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Some Combinatorial Results on Star-Like Transformation Semigroup $T\alpha\omega_n^*$

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Abstract

Let X_n be a finite set and star-like full transformation semigroup $T\alpha\omega_n^*$ be a semigroup of Full Transformation semigroup T_n of X_n. Let Height of α^* be $H^+(\alpha^*) = \mid Im\alpha^* \mid$, Fixed point of α^* be $F(\alpha^*) = \mid \{x \in X : x\alpha^* = x\} \mid$, Idempotent of α^* be $\mid F(\alpha^*) \mid = \mid Im\alpha^* \mid$, Collapse of (α^*) be $\mid \cup\{t\alpha^{-1} : t \in T\alpha\omega^*\} \mid$ and Relapse of (α^*) be $\mid n - C^+((\alpha^*)) \mid$ and also, Green's relation of semigroup $T\alpha\omega_n^*$ were characterized using the general method and definitions.The methods employed in carrying out this research work were that the elements in each of the functions were listed and some tables were formed for $H^+(\alpha^*)$, $J^*(\alpha^*)$, $E \mid q^*(\alpha^*) \mid$, $C^*(\alpha^*)$, $C^-(\alpha^*)$ and $\mathcal{L}$, $\mathcal{R}$,$\mathcal{D}$, $\mathcal{H}$ and $\mathcal{J}$ equivalence relations from these tables, triangular array and sequences were formed; the patterns of the arrangement were studied, formulae were deduced in different cases through the combinatorial principle.

1. Introduction

The full transformation semigroup denoted as T_n given $X_n = \{1, 2, 3, ..., n\}$ such that α:Dom $\alpha = X_n$, commonly known has full or total transformation semigroup with set S and operation $*$ Transformation semigroup are associative then:(α, β, μ):$(\alpha * \beta) * \mu$=$\alpha * (\beta * \mu)$. It is also known as the full symmetric semigroup or monoid with composition of mappings as the semigroup operator. The star-like full transformation semi-group denoted as $T\alpha\omega_n^*$ in T_n and also a semigroup in full transformation semigroup.

A Star-like transformation semigroup is said to satisfy collapse function if $c^+(\alpha^*) = \mid \bigcup t\alpha^{-1} : t \in T\alpha\omega_n^* \mid$ while Relapse function is denoted as $r^+(\alpha) =\mid n - c^+(\alpha^*) \mid$ where $n \in N$. The Green's relation are useful for understanding the nature of divisibility in a semigroup, instead of working directly with a semigroup S, it is current to define Green's

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relation over the monoid S'. S' is S with an identity adjoined if S is not already a monoid [9], a new element is adjoined and defined to be an identity. Let S' be a semigroup, $a, b \in S$. If a and $b$generate the same left principal ideal, that is, $S'a = S'b$, then we say that a and b are L equivalent and write a $\mathcal{L}$ b or $(a, b) \in \mathcal{L}$. If a and b generate the same right principal ideal, that is, $a^{'} = b^{'}$, then we say that a and b are $\mathcal{R}$ equivalent and write a$\mathcal{R}$ b or $(a, b) \in \mathcal{R}$. If a and b generate the same principal ideal, that is, $S^{'}aS^{'} = S^{'}bS^{'}$, then we say that a and b are $\mathcal{J}$ equivalent and write a$\mathcal{J}$b or $(a, b) \in \mathcal{J}$. Let $\mathcal{H} = \mathcal{L} \bigcap \mathcal{R}$, $\mathcal{D} = \mathcal{L} \cup \mathcal{R}$, then $\mathcal{H}, \mathcal{D}$ are equivalences on S, too. It is a well known fact that $\mathcal{J} = \mathcal{D}$ in any finite semigroup. These five equivalences are usually called Green$^{'}$s equivalence relations on **S**. Researchers have contributed to the study of contraction mapping and its relation to semigroup. [11], established properties of ω-order preserving partial contraction mapping and its relation of Co-Semigroup. [4] established the multiplicative invertibility characterization on star-like cyclicpoid $C_yPW_n^*$ finite partial transformation semigroup. However, The basis of this research is to investigate results of collapse, relapse, idempotent, height, fixed point and Green$^{'}$s Relation on star-like full transformation semigroup $T\alpha\omega_n^*$ and to generalize the sequences obtained. Algebraic and combinatorial properties of the transformation semigroups have been studied over time and some interesting results have emerged from the study of some notable Mathematicians such as: [5], [6], [10], [7]. Their study has resulted into useful tools which have been applied on various aspects of combinatorial mathematics. In fact, the study of semigroups of full T_n, partial P_n and partial one-one I_n has been fruitful over the years. It is worth to mention that this paper was inspired by the study of [3], [10]. [8] explained some combinatorial results on green's relation of partial injective transformation semigroup and characterized the Green$^{'}$s relation on CI_n and also solved the contraction mapping injective partial transformation semigroup on $n - objects$. Using two parameters $F(n, p)$ and found that the order of $\mathcal{L}$- classes and $\mathcal{R}$ class are the same but $\mathcal{D}$ class is different.

2. Preliminary Notes

For the purpose of completeness we give some basic definitions that we shall need in the coming sections.

Definition 2.1. Star-like Semigroup Let $X_n = \{1, 2, 3, ..., n\}$ be finite n order non- negative integers, then a finite semigroup is said to be star-like if $\mid K_{i+1} - \lambda K_i \mid \leq \mid K_i - \lambda K_{i+1} \mid$ where $i \in N$ such that $N \cup 0 \in \Re$ for all $K_i\omega_i \in \alpha\omega_n^*$. Akinwunmi, Mogbonju and Adeniji (2021).

Definition 2.2. The cardinality $|r_n^+(\alpha^*)|$ of an image of a star-like semigroup $T\omega_n^*$ is called the height of the semigroup. It is the number of different elements in the image sets of α_n^* such that $rank\ (\alpha^*) = |I(\alpha^*)|$ which is denoted by $rank\ (\alpha^*) = |r_n^+(\alpha^*)|$.

Definition 2.3. [1]: In a Transformation Semigroup S, an element α in S is collapsible, $c(\alpha)$ if there exists a number $C^+(\alpha) =\mid t\alpha^{-1} \mid ?2$ where t is an element in the image of α.

Definition 2.4. [2]: An element $\epsilon \in S$ is an idempotent if $\epsilon^2 = \epsilon$, a full transformation ϵ is idempotent if and only $Im\epsilon = F(\epsilon)$ Where $F(\epsilon)$ is the set of all fixed point of the transformation and $Im(\epsilon)$ is the image set of the Transformation. The set of all idempotent of S is denoted by $E = E(S)$.

Definition 2.5. The Fix of a transformation α is define and denoted by $f(\alpha) = \mid F(\alpha) \mid = \mid x \in Dom\alpha : x\alpha = x \mid$.

The full transformation semigroup elements were derived using the general formular $n^n \forall n \geqslant 1$ while the star-like semigroup elements were extracted from the full transformation semigroup elements using the star-like operator.

$$| K_{n+1} - \lambda K_n | \leq | K_n - \lambda K_{n+1} |, \tag{2.1}$$

where $n \in \mathbb{N}$ the element were arranged accordingly with respect to $\{1, 2, 3, \cdots, n\}$ using the elements generated to investigate for functions such as Height $H^+(\alpha\omega_n^*)$, Fixed point $J^*(\alpha\omega_n^*)$, Idempotent $Eq^*(\alpha\omega_n^*)$, Collapse $C^+(\alpha\omega_n^*)$, Relapse $C^-(\alpha\omega_n^*)$ and the five (5) equivalence classes in Green$'$s relations ($\mathcal{L}$, $\mathcal{R}$, $\mathcal{D}$, $\mathcal{H}$, $\mathcal{J}$).
From these tables triangular array and sequences were formed, the pattern of arrangement were studied and general formulars were generated in different cases through combinatorial principles.

let $| K_{n+1} - \lambda K_n | \leq | K_n - \lambda K_{n+1} |$ such that $n \in \mathbb{N}$
when $n = 1$

$$T\alpha\omega_1^* = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$T\alpha\omega_1^*$ has 1 element.
when $n = 2$

$$T\alpha\omega_2^* = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$$

$T\alpha\omega_2^*$ has 3 elements.

when $n = 3$

$$T\alpha\omega_3^* = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 3 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \end{pmatrix}$$

$T\alpha\omega_3^*$ has 10 elements.

3. Green's Equivalence Relation of Star-like Full Transformation

Table 1 : For $T\alpha\omega_1^*$ on $X_1 = \{1\}$ = 1 element: $n = 1$ $r = 1$

$\frac{ker(\alpha)}{Im(\alpha)}$	$\{1\}$
1	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Table 2 : For $T\alpha\omega_2^*$ on $X_2 = \{1, 2\}$ = 3 elements.
$n = 2$ $r = 1$

$\frac{ker(\alpha)}{Im(\alpha)}$	$\{2\}$
1,2	$\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$

Table 3 : n = 2 r = 2

$\frac{ker(\alpha)}{Im(\alpha)}$	$\{1,2\}$
1/2	$\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}\begin{pmatrix}1 & 2\\2 & 1\end{pmatrix}$

4. Main Results

The results obtain in this research work shows the classical features in the application of transformation semigroup with combinatorial properties. We exhibit some of the techniques that was used to obtain these results, such that for any transformation in $T\omega_n^*$, $\Delta^*_{(i,j)}$ is the operator order for all vertical elements of $\alpha^* \in T\omega_n^*$, and $\nabla^*_{(i,j)}$ is the operator order for all diagonal elements of $\alpha^* \in T\omega_n^*$.

Proposition 4.1. *Let $\delta^* \in T\ \alpha\omega_n^*$ be a star-like transformation, then*

$$3 \mid T\alpha\omega_n^* \mid = \frac{\binom{13n^4}{13p^4 - b} - 23\binom{5n^3}{5p^3 - b}}{2} + 5\binom{38n^2}{38p^2 - b} - 7\binom{37n}{37p - b} + 123$$

such that $n = p \geq 1$ where b is a star-like algebraic constant.

Proof. Suppose $N_i = \{i, i+1, i+2, i+3, \cdots, n\}$, $i = \{0, 1, 2, ...\}$ is non-negative with $N_0 = 0, 1, 2, ...$ if $\delta^* \in$ T $\alpha\omega_n^*$ is a star- like transformation with $P^* \leq 1$ and $P^* \geq 1$ there exist some star-like sequences U_n with (ς_n^* is vertical order)
$\varsigma_n^* = \mid \delta^* \in$ T $\alpha\omega_n^* \mid$, then

$$U_n = \varsigma_n^{*1}\binom{n}{1} + \varsigma_n^{*2}\binom{n}{2} + \varsigma_n^{*3}\binom{n}{3} + \cdots + \varsigma_n^{*k+1}\binom{n}{k}$$

posses a unique integer difference of 52 at ς_n^{*4}, for all $i \geq n \geq 1$. we generate a system of equation.

$U_0 + U_1 + U_2 + U_3 + U_4 = 1$

$16U_0 + 8U_1 + 4U_2 + 2U_3 + U_4 = 3$

$81U_0 + 27U_1 + 9U_2 + 3U_3 + U_4 = 10$

$256U_0 + 64U_1 + 16U_2 + 4U_3 + U_4 = 37$

$625U_0 + 125U_1 + 25U_2 + 5U_3 + U_4 = 151$

Since δ is a bijective mapping,the system of equation may be re-written as $\mathbf{A\ U = X}$ such that

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & :1 \\ 16 & 8 & 4 & 2 & :1 \\ 81 & 27 & 9 & 3 & :1 \\ 256 & 64 & 16 & 4 & :1 \\ 625 & 125 & 25 & 5 & :1 \end{pmatrix}, U = \begin{pmatrix} U_0 \\ U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix}, X = \begin{pmatrix} 1 \\ 3 \\ 10 \\ 37 \\ 151 \end{pmatrix}$$

using Maple 18, we obtained

$$U_0 = \frac{13}{6}, U_1 = \frac{-115}{6}, U_2 = \frac{190}{6}, U_3 = \frac{-518}{6}, U_4 = 41$$

We see that ς_n^* gives the required star-like recursive relation of $T\alpha\omega_n^*$ for $P^* \leq 1$, and $P^* \geq 1$.
The result follows in TABLE $\{5, 6, 8 and 9\}$

□

Theorem 4.2. *Given any star-like transformation δ^* in full star-like semigroup $T\alpha\omega_n^*$, then the following statement are equivalent:*

i. Any $\delta^* \in T\ \alpha\omega_n^*$ has a maximum element $m(\delta^*)$

ii.

$$\mid \delta^* T\alpha\omega_n^* \mid = \sum_{q^*=1}^{n} \binom{n+2q^*}{3q^*}$$

.

iii.

$$F(n, c^* J^*) = \binom{2^{(n-c^*)+1}}{2^{(n-J^*)+1} - 1}; n \geq c^* \geq J^* \geq q^* \geq 1$$

.

Proof. $(i) \implies (ii)$ suppose $F(n, c^* J^*)$ is a star-like composite function, let $N_i = \{i, i+1, i+2, i+3, \cdots, n\}$, $i = \{0, 1, 2, ...\}$ be distinct non - negative integer such that $T\alpha\omega_n^*$ contains a star - like transformation $\delta^* \in$ T $\alpha\omega_n^*$ then $i \in N_i$, the $Dom(\delta^*)$ can be chosen from $N_i in \binom{q^*}{n}$ if δ^* is a bijective map and $T\alpha\omega_n^*$ is reducible Im$(\delta^*) = 0$ if $\mid q^*(\delta^*) \mid = 1$ but if $n = q^*$; $\mid q^*(\delta^*) \mid \neq$ -1 for each value of $\delta^* T\alpha\omega_n^*$. such that
$\mid T\alpha\omega_n^* \mid = \binom{n+2q^*}{3q^*}$

$(ii) \implies (iii)$ since $\delta^* \in$ T $\alpha\omega_n^*$ is star-like, there exist finitely many element such that $Dom(\delta^*) = Im(\delta)$ where $\rho_n^* \cap \varsigma_n^*$ form a total

$$\sum_{q^*=1}^{k} \binom{2^{(n-q)+1}}{2^{(n-m)+1} - 1} \binom{n+2q^*}{3q^*} = F(n, q, J)$$

by proposition 1, ς_n^* and ρ_n^* are vertical and diagonal difference operator order of element in $\delta^* T\alpha\omega_n^*$.

$(iii) \implies (i)$ suppose $m(\delta^*)$ denote maximum element in $Im(\delta^*)$ and $T\alpha\omega_n^* \subseteq \alpha\omega_n^*$, then consider a mapping

$$\gamma^* = \begin{pmatrix} u_2 & u_1 & u_0 \\ m_2 & m_1 & m_0 \end{pmatrix}$$

; $\in m(\delta^*)$
such that there exist another element $\leftthreetimes^* \in m(\delta^*)$ with $\gamma^* \leq \leftthreetimes^*$ and $I_0 < \leftthreetimes^*$. since γ^* and $\leftthreetimes_0$ are bijective, we have

$(\gamma T\alpha\omega_n^*) \cap (\leftthreetimes_0 T\alpha\omega_n^*) = m(\delta^*)$

if $u_i = u_1 u_0 = u_0 \leftthreetimes^* = m_0 \gamma^*$, then $\mid T\alpha\omega_n^* \mid = 1$ Thus, the result follows in Tables 6 and 7. □

Theorem 4.3. *Let $\delta^* \in T\alpha\omega_n^*$ be a star-like transformation, if $\mid b_n \mid$ denote the cardinality of maximum fixed element in $Dom\delta^*$, there exist a non- negative integer k such that ρ_n^{k+1} = 0, then*

$$F(n, J^*, e^*) = \binom{2J^* - e^0}{2n - 2^2};$$

Proof. Suppose $\rho_n^* b_n = 0$ is the diagonal star-like operate of the maximal element of $\delta^* \in T\alpha\omega_n^*$ with e^0 as a star-like exponential constant. If $u_1, u_2, ...u_n$ are star-like sequences of number in which there exist a non-negative integer k for all $n \geq 2$ in $n \in N_i = \{i, i+1, i+2, ...\}$ such that $\rho_n^{k+1} = 0$ by Theorem 2, $\delta^* \in T\alpha\omega_n^*$ can be expressed as a reducible star-like polynomial and since δ^* is bijective under composition of mapping, we see that $f(J^*)$ can be chosen in $\binom{J}{n-1}$; ways for all $J \geq 1$, such that

$$F(n, J^* e^*) = \binom{2J^* - e^0}{2n - 2^2};$$

The result follows in Table 6. □

Lemma 4.4. *Let $T\alpha\omega_n^* \subseteq P\alpha\omega_n^* \subset \alpha\omega_n^*$ be a star-like full finite semigroup, given any $\rho^* \in T\alpha\omega_n^*$,*

$$F(n, m^*) = \binom{2_n - 1}{n - m^*}; m^* \geq n \geq 1$$

Proof. Suppose $\rho^* \in T\alpha\omega_n^*$ such that $n \in N_i = \{i, i+1, i+2, ...\}$ (i = $\{0, 1, 2, ...\}$), then $m^*(\alpha)$ has an identity element **e** in which m^* is bijective under composition of mapping with $m^* \in Dom(\alpha)$. Each element of $Dom(\alpha)$ in $T\alpha\omega_n^*$ can be chosen from N_i in

$$F(n, m^*) = \binom{2_n - 1}{n - m^*}; ways$$

□

Lemma 4.5. *Let $ET\alpha\omega_n^*$ be star-like idempotent semigroup, if $\delta^* \in T\alpha\omega_n^* \subseteq ET\alpha\omega_n^*$ then* $\mid \varsigma_k^* T\alpha\omega_n^* \mid =$

$$\frac{\binom{k^2 - k}{k^2 - (k+1)}}{m!}; m = 2; k \geq 2.$$

Proof. Suppose $ET\alpha\omega_n^* \subseteq T\alpha\omega_n^*$ with a fixed point value of order 2, such that $f(J^*) = 2$. Given any star-like transformation $\delta^* \in T\alpha\omega_n^* \subseteq ET\alpha\omega_n^*$ that satisfy the star-like idempotency structure. we have (i) $\gamma^*.\gamma^* = \gamma^*$ (ii) $f(m^*) = Im(\gamma^*)$ Let ς_n^* be a star-like vertical difference operator of $ET\alpha\omega_n^*$ under usual star-lke composition of mapping with a fixed point of $m^* = 2$ we generate a system of equation.

$U_3 + U_2 + U_1 + U_0 = 1$

$8U_3 + 4U_2 + 2U_1 + U_0 = 3$

$27U_3 + 9U_2 + 3U_1 + U_0 = 6$

$64U_3 + 16U_2 + 4U_1 + U_0 = 10$

Since δ is a bijective mapping,the system of equation may be re-written as $\mathbf{A}\ \mathbf{U} = \mathbf{X}$ such that

$$A = \begin{pmatrix} 1 & 1 & 1 & :1 \\ 8 & 4 & 2 & :1 \\ 27 & 9 & 3 & :1 \\ 64 & 16 & 4 & :1 \end{pmatrix}, U = \begin{pmatrix} U_0 \\ U_1 \\ U_2 \\ U_3 \end{pmatrix}, X = \begin{pmatrix} 1 \\ 3 \\ 6 \\ 10 \end{pmatrix}$$

using Maple 18, we obtained

$$U_0 = 0, U_1 = \frac{1}{2}, U_2 = \frac{1}{2}, U_3 = 0$$

we see from Theorem 2 and Lemma 2 that

$\mid \varsigma_k^* T\alpha\omega_n^* \mid =$

$$\frac{\begin{pmatrix} k^2 - k \\ k^2 - (k+1) \end{pmatrix}}{m!}$$

; $k \geq 2$; $n \in N_i = \{i, i+1, i+2, ...\}$

gives a star-like recursive relation of $\varsigma_k^* \subseteq \gamma^*$ for a complete star-like triangular array of $ET\alpha\omega_n^*$ Hence, the result

A system of equation.

$U_4 + U_3 + U_2 + U_1 + U_0 = 1$

$16U_4 + 8U_3 + 4U_2 + 2U_1 + U_0 = 2$

$81U_4 + 27U_3 + 9U_2 + 3U_1 + U_0 = 5$

$256U_4 + 64U_3 + 16U_2 + 4U_1 + U_0 = 13$

$625U_4 + 125U_3 + 25U_2 + 5U_1 + U_0 = 33$

Since δ is a bijective mapping,the system of equation may be re-written as $\mathbf{A}\ \mathbf{U} = \mathbf{X}$ such that

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & :1 \\ 16 & 8 & 4 & 2 & :1 \\ 81 & 27 & 9 & 3 & :1 \\ 256 & 64 & 16 & 4 & :1 \\ 625 & 125 & 25 & 5 & :1 \end{pmatrix}, U = \begin{pmatrix} U_0 \\ U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix}, X = \begin{pmatrix} 1 \\ 2 \\ 5 \\ 13 \\ 33 \end{pmatrix}$$

using Maple 18, we obtained

$$U_0 = 3, U_1 = \frac{-29}{6}, U_2 = \frac{23}{6}, U_3 = \frac{-7}{6}, U_4 = \frac{1}{6}$$

with general equation of
$U_n = \frac{1}{6}n^4 - \frac{7}{6}n^3 + \frac{23}{6}n^2 - \frac{29}{6}n + 3$
Recursive Formular

$$F(n,q^*) = \frac{\binom{n^4}{p^4-k} - \binom{3!+1}{3!}n^3 + \binom{4!-1}{4!-2}n^2 - \binom{4!+5}{4!+4}n + 3!.3}{3!}$$

□

Theorem 4.6. *Suppose $\delta^* \in T\ \alpha\omega_n^*$ be a star-like transformation, then*

$$F(n,\mathcal{L}) = \binom{x^n}{x^p - k}$$

such that $n = p \geq 0$ where k is a star-like algebraic constant.

Proof. Let $N_i = \{i, i+1, i+2, i+3, \cdots, n\}$, $i = \{0,1,2,...\}$ is non-negative with $N_0 = 0,1,2,...$ since $\delta^* \in$ T $\alpha\omega_n^*$ is a star- like transformation with $n \geq 0$ and $P \geq 1$ there exist some star-like sequences N_n with ς_n^* is vertical order which posses a unique integer difference of 1 at ς_4^* for all $i \geq n \geq 0$
$\varsigma_n^* = |\ \delta^* \in$ T $\alpha\omega_n^*\ |$, then
System of equation for $\mathcal{L}$

$$a_4 + a_3 + a_2 + a_1 + a_0 = 1$$

$$16a_4 + 8a_3 + 4a_2 + 2a_1 + a_0 = 2$$

$$81a_4 + 27a_3 + 9a_2 + 3a_1 + a_0 = 4$$

$$256a_4 + 64a_3 + 16a_2 + 4a_1 + a_0 = 8$$

$$625a_4 + 125a_3 + 25a_2 + 5a_1 + a_0 = 16$$

Using maple 18, The following were obtained
$a_n = \frac{1}{24}n^4 - \frac{1}{4}n^3 + \frac{23}{24}n^2 - \frac{18}{24}n + 1$

Recursive Formular

$$F(n,\mathcal{L}) = \binom{x^n}{x^p - k}$$

such that x = 2 and n,p = $\{0,1,2,3,...\}$ k= Algebraic constant. □

Lemma 4.7. *For any transformation α^* in $T\omega_n^*$ there are finitely many star-like $\alpha^* \in T\omega_n^*$ such that:*

$$F(n; r_n^+(\alpha^*), m(\alpha^*)) = \begin{pmatrix} 2(n-2) + (n - m(\alpha^*)) \\ n - m(\alpha^*) \end{pmatrix}$$

for all $r_n^+(\alpha^*) \geq m \geq 1$ and $n \in \mathbb{N}$.

Proof. Suppose $\alpha^* \in T\omega_n^*$ then if α^* is a bijection and under composition of mapping where $F(n; r_n^+(\alpha^*), m(\alpha^*))$ is reducible then by definition we observe that

$$F(n; r_n^+(\alpha^*), m(\alpha^*)) = \begin{pmatrix} 2(n-2) + (n - m(\alpha^*)) \\ n - m(\alpha^*) \end{pmatrix}$$

such that $\Delta^*_{n,r_n^+(\alpha^*)} = \nabla^*_{(n,m)}$ where $r_n^+(\alpha^*) \geq m \geq 1$ for all $n \in \mathbb{N}$. Thus, the result follows immediately from table (2) and (3). □

Theorem 4.8. *Let $\mathcal{R} \in T\alpha\omega_n^*$ then $a\mathcal{R}b$ if and only if ker(a) = ker(b)and also $\mathcal{H} \in T\alpha\omega_n^*$ such that $\mathcal{H} = im(\alpha) = im(\beta), ker(a) = ker(a)$ then the following results were obtained*

i

$$F(n,\mathcal{R}) = \frac{\binom{14n^4}{14p^4-k} - \binom{418n^2}{418p^2-k}}{4!} - \frac{\binom{31n^3}{31p^3-k} - \binom{143n}{143p-k}}{3!} + 12$$

ii

$$F(n,\mathcal{H}) = \frac{\binom{115n^4}{115p^4-k} + \binom{3629n^2}{3629p^2-k}}{4!} - 2\frac{\binom{537n^3}{537p^3-k} + \binom{2535n}{2535p-k}}{4!} + 101$$

Proof. Since $a\mathcal{R}b$ is ker(a) = ker(b) then the sequence generated are 1, 2, 5, 14 & 47 [12] and system of equation for $\mathcal{R}$ is given below.

$a_4 + a_3 + a_2 + a_1 + a_0 = 1$

$16a_4 + 8a_3 + 4a_2 + 2a_1 + a_0 = 2$

$81a_4 + 27a_3 + 9a_2 + 3a_1 + a_0 = 5$

$256a_4 + 64a_3 + 16a_2 + 4a_1 + a_0 = 14$

$625a_4 + 125a_3 + 25a_2 + 5a_1 + a_0 = 47$

Using maple 18, The following were obtained

$$a_n = \frac{7}{12}n^4 - \frac{31}{6}n^3 + \frac{209}{12}n^2 - \frac{143}{6}n + 12$$

Recursive Formula

$$F(n,\mathcal{R}) = \frac{\binom{14n^4}{14p^4-k} - \binom{418n^2}{418p^2-k}}{4!} - \frac{\binom{31n^3}{31p^3-k} - \binom{143n}{143p-k}}{3!} + 12$$

while System of equation for $\mathcal{H}$

$a_4 + a_3 + a_2 + a_1 + a_0 = 1$

$16a_4 + 8a_3 + 4a_2 + 2a_1 + a_0 = 2$

$81a_4 + 27a_3 + 9a_2 + 3a_1 + a_0 = 8$

$256a_4 + 64a_3 + 16a_2 + 4a_1 + a_0 = 38$

$625a_4 + 125a_3 + 25a_2 + 5a_1 + a_0 = 226$

Using maple 18, The following were obtained

$$a_n = \frac{115}{24}n^4 - \frac{537}{12}n^3 + \frac{3629}{24}n^2 - \frac{2535}{12}n + 101$$

Recursive Formular

$$F(n,\mathcal{H}) = \frac{\begin{pmatrix} 115n^4 \\ 115p^4 - k \end{pmatrix} + \begin{pmatrix} 3629n^2 \\ 3629p^2 - k \end{pmatrix}}{4!} - 2\frac{\begin{pmatrix} 537n^3 \\ 537p^3 - k \end{pmatrix} + \begin{pmatrix} 2535n \\ 2535p - k \end{pmatrix}}{4!} + 101$$

Recursive Formular For $\mathcal{D}$

$\mathcal{D}$ = n where n = $\{1, 2, 3, 4....\}$ The results follows in Tables 11 and 13. □

Proof. Suppose $\alpha \in T\omega_n^*$ such that $M = \{w_1, w_2, \ldots\}$ then $m(\alpha^*)$ has an identity element e in which $m(\alpha^*)$ is bijection under composition of mapping such that $m(\alpha^*) \in D(\alpha^*)$. Then the each element $D(\alpha^*)$ of $T\omega_n^*$ can be chosen from $M = \{w_1, w_2, \ldots\}$ in $\begin{pmatrix} 2m(\alpha^*) - 1 \\ n - m(\alpha^*) \end{pmatrix}$ ways. Which is equivalent to $2m - 1$ if $m = n - 1$ for all $m(\alpha^*) \geq 1$; $n \geq 2$ where $m, n \in \mathbb{N}$. The result follow immediately from table (3). □

Table 4: HEIGHT TABLE $H^+(\alpha^*)$

$H^+/\alpha\omega_n^*$	1	2	3	4	5	$\sum H^+(\alpha\omega_n^*)$
1	1					1
2	1	2				3
3	1	7	2			10
4	1	16	18	2		37
5	1	33	84	31	2	151

Table 5: FIXED POINT TABLE $J^*(\alpha^*)$

$J^*/\alpha\omega_n^*$	0	1	2	3	4	5	$\sum J^*(\alpha\omega_n^*)$
1	-	1					1
2	1	1	1				3
3	3	3	3	1			10
4	12	9	10	5	1		37
5	42	38	40	23	7	1	151

Table 6: IDEMPOTENT TABLE $E \mid q^*(\alpha\omega_n^*) \mid$

$n/E \mid q^*(\alpha\omega_n^*) \mid$	1	2	3	4	5	$\sum E \mid q^*(\alpha\omega_n^*) \mid$
1	1					1
2	1	1				2
3	1	3	1			5
4	1	6	5	1		13
5	1	10	14	7	1	33

Table 7: COLLAPSE TABLE $C^{+}(\alpha\omega_n^*) \geq 2$

$n/C^{+}(\alpha\omega_n^*$	0	1	2	3	4	5	$\sum C^{+}(\alpha\omega_n^*)$
1	1	-					1
2	2	-	1				3
3	2	-	7	1			10
4	2	-	19	9	7		37
5	2	-	31	38	59	21	151

Table 8: RELAPSE TABLE $C^{-}(\alpha\omega_n^*)$

$n/C^{-}(\alpha\omega_n^*)$	0	1	2	3	4	5	$\sum C^{-}(\alpha\omega_n^*)$
1	-	1					1
2	1	-	2				3
3	1	7	-	2			10
4	7	9	19	-	2		37
5	21	59	38	31	-	2	151

Table 9: $\mathcal{L}$ - Classes of $T\alpha\omega_n^*$

n/r	1	2	3	4	5	$\sum\mathcal{L}$
1	1					1
2	1	1				2
3	1	2	1			4
4	1	3	3	1		8
5	1	4	6	4	1	16

Table 10: $\mathcal{R}$ - Classes of $T\alpha\omega_n^*$

n/r	1	2	3	4	5	$\sum\mathcal{R}$
1	1					1
2	1	1				2
3	1	3	1			5
4	1	7	5	1		14
5	1	15	22	8	1	47

Conclusion: In this manuscript, we investigated the star-like semigroup elements from full transformation semigroup elements and the combinatorial results of different functions were also considered. We conclude here that star-like full transformation semigroup has shown to be an encouraging area to study in theory of transformation semigroup.

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