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APPLICATION OF HOMOTOPY PERTURBATION METHOD (HPM) FOR THE SOLUTION OF NANOFLUIDS OVER A POROUS STRETCHING SURFACE

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Abstract

In this study, by means of homotopy perturbation method (HPM) and sixth order Runge-Kutta method an approximate analytical and numerical solutions of the of viscous ag-water and Cu-water nanofluids over a porous stretching surface is obtained. Graphical results are presented to investigate the influence of the nanoparticles volume fraction parameter and suction/injection parameter on the velocity and skin friction. The validity of our solutions is verified by the previous work.

1. Introduction

Most of the scientific problems and phenomena are modeled by nonlinear ordinary or partial differential equations. In recent years, many powerful methods have been developed to construct explicit analytical solution of nonlinear differential equations. Among them, two analytical methods have drawn special attention, namely, the homotopy perturbation method HPM [1-2] and homotopy analysis method HAM [3-6]. The essential idea in these methods is to introduce

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a homotopy parameter, say p, which takes the value from 0 to 1. For $p = 0$, the system of equations takes a simplified form which readily admits a particularly simple solution. When p is gradually increased to 1, the system goes through a sequence of deformations, the solution of each of which is close to that at the previous stage of deformation. Eventually at $p = 1$, the system takes the original forms of equation, and the final stage of deformation gives the desired solution. We know that all perturbation methods require small parameter in nonlinear equation, and the approximate solutions of equation containing this parameter are expressed as series expansions in the small parameter. Selection of small parameter requires a special skill. A proper choice of small parameter gives acceptable results, while an improper choice may result in incorrect solutions. The homotopy perturbation method, which is a coupling of the traditional perturbation method and homotopy in topology, does not require a small parameter in equation modeling phenomena. In recent years, the HPM has been successfully employed to solve many types of linear and nonlinear problems [7-10].

Nanofluids are defined as dilute suspension containing tiny particles having diameter less than 100 nm. Choi [11] experimentally verified that addition of small amount of nanoparticles appreciably enhances the effective thermal conductivity of the base fluid. These particles are made up of metals such as (Al, Cu), oxides (Al_2O_3), carbides (SiC), nitrides (AlN, SiN) or nonmetals (graphite, carbon nanotubes). Buongiorno [12] proposed a mathematical model that considered two significant effect namely the Brownian motion and thermophoretic diffusion of nanoparticles. Kuznetsov and Nield [13] numerically studied the flow of nanofluid past a vertical flat plate. The ChengMinkowcz problem for natural convective boundary layer flow of a nanofluid occupying a porous space was considered by Nield and Kuznetsov [14]. Similar attempts in this direction include those of Nield and Kuznetsov [15,16] and Kuznetsov and Nield [17]. The boundary layer flow of nanofluid over a continuously moving surface with a parallel free stream has been studied by Bachok et al. [18]. Khan and Pop [19] provided numerical solutions for boundary-layer flow of nanofluid over a stretching sheet. Rana and Bhargava [20] numerically investigated the flow of nanofluid over a nonlinearly stretching sheet by finite element method (FEM). Makinde and Aziz [21] examined the flow of nanofluid over a stretching sheet in the presence of convective surface boundary conditions. Unsteady boundary layer flow of nanofluid over a stretching/shrinking sheet has been examined by Bachok et al. [22].

The flow of viscous Ag-water and Cu-water nanofluids over a stretching surface have not been investigated using HPM, although the use of Keller Box Method of solution is known for such problems (Vajravelu et. al. [23]).

Hence, the purpose of this paper is to apply HPM to obtain the approximate analytical solution to the problem of viscous Ag-water and Cu-water nanofluids flow over a stretching sheet. It went further to compare the result obtained with that of sixth order Runge-Kutta method for the purpose of analysis.

2. Equation of motion

Consider a steady two-dimensional laminar fluid flow over a stretching sheet. The basic boundary layer equation can be written as [23],

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

where u and v are the velocity components along x and y axis respectively. The boundary conditions for the problem are

$$u = bx,\ v = V_0 \text{ at } y = 0\ u \longrightarrow 0,\ \text{as } y \longrightarrow \infty \tag{3}$$

The effective density of nanofluids is given as

$$(\rho)_{nf} = (1-\phi)\rho_f + \phi\rho_s \tag{4}$$

where ϕ is the solid volume fraction of nanoparticles. while the effective dynamic viscosity of the nanofluid given by Brinkman [19] as

$$\mu_{nf} = \frac{\mu_{nf}}{(1-\phi)^{2.5}} \tag{5}$$

Here the subscripts nf, f and s represent respectively the thermo-physical properties of the nanofluids, base fluid and the nano-solid particles.
By introducing the similarity transformation

$$\eta = \sqrt{\frac{b}{v_f}}y,\ u = bxf'(\eta),\ v = -\sqrt{bv_f}f(\eta) \tag{6}$$

where prime denotes differentiation with respect to η. Substituting Eq.(6) into Eq.(2), the governing equation and boundary conditions reduce to

$$f''' = (1-\phi)^{25}\left[(1-\phi) + \phi\left(\frac{\rho_s}{\rho_f}\right)\right]\left\{f'^2 - ff''\right\} \tag{7}$$

$$f = -R,\ f' = 1 \text{ at } \eta = 0,\ f' \longrightarrow 0,\ \text{as } \eta \longrightarrow \infty \tag{8}$$

here $R = \frac{V_0}{\sqrt{vb}}$, where $R > 0$ corresponds to suction and $R < 0$ for injection. Eq (7) is nonlinear differential equation which can be solve analytically by HPM.

3. Method of Solution

We will use HPM in order to obtain the solution of Eq.(7). Assuming $f = \phi$ Eq.(7) can be written in following form

$$\varphi''' + F(\varphi) = 0 \tag{9}$$

in which

$$F(\varphi) = -(1-\phi)^{2.5}A(\varphi'^2(\eta) - \varphi(\eta)\varphi''(\eta)) \tag{10}$$

where

$$A = (1-\phi) + \phi(\frac{\rho_s}{\rho_f}) \tag{11}$$

Let

$$A^* = (1-\phi)^{2.5}A \tag{12}$$

According to the homotopy perturbation method [25], we construct a homotopy in the form

$$\varphi''' - \alpha^2\varphi' + p(F(\varphi) + \alpha^2\varphi') = 0 \tag{13}$$

with the initial conditions

$$\varphi'(0) = 1,\ \varphi(0) = R,\ \varphi^{'}(\infty) = 0 \tag{14}$$

When $p = 0$ (13) becomes a linearised equation $\varphi''' - \alpha^2\varphi' = 0$ where α is an unknown parameter to be further determined. When $p = 1$, the equation becomes the original problem. The embedded parameter p monotonically increases from zero to unit as the trivial problem, $\varphi''' - \alpha^2\varphi' = 0$, is continuously deforms to the original problem, Eq. (9). By introducing the HPM, we assume that the solution to (13) can be written as a power series in p.

$$\varphi = \varphi_0 + p\varphi_1 + p^2\varphi_2 + ... \tag{15}$$

Substituting (15) into (13) and equating the terms with the identical power of p, we have

$$p^0 : \varphi_0''' - \alpha^2\varphi_0' = 0,\ \varphi_0'(0) = 1,\ \phi_0(0) = -R,\ \phi_0'(\infty) = 0 \tag{16}$$

$$p^1 : \varphi_1''' - \alpha^2\varphi' + \alpha^2\varphi' + F(\varphi),\ \phi_1'(0) = 0,\ \phi_1'(0) = 0,\ \phi_1'(\infty) = 0 \tag{17}$$

where

$$F(\varphi) = A^*(\phi_0\varphi_0'' - \varphi_0'^2) \tag{18}$$

The solution of (16) and (17) can be readily obtained, which reads

$$\varphi_0 = -R + \frac{1}{\alpha}(1 - e^{a\eta}) \tag{19}$$

$$\varphi_1 = -\frac{(\alpha^2 + A^*(R\alpha - 1))}{2\alpha^2}\eta e^{-\alpha\eta} \tag{20}$$

where

$$\alpha = \frac{1}{2}\left[-A^*R + \sqrt{(A^*R)^2 + 4A^*}\right] \tag{21}$$

Therefore, we obtain

$$\varphi(\eta) = \varphi_0(\eta) + \varphi_1(\eta) \tag{22}$$

Setting $\phi = 0$ and $R = 0$, the present problem returns to the flow problem studied by Crane. The exact solution for the velocity field is

$$f(\eta) = 1 - e^{-\eta} \tag{23}$$

in the presence of nanofluid particles volume fraction ($\phi \neq 0$) the exact solution to Eq.7 satisfying the required boundary conditions is given by

$$f(\eta) = e^{-\alpha\eta} \tag{24}$$

where

$$\alpha = \frac{1}{2}\left[-A^*R + \sqrt{(A^*R)^2 + 4A^*}\right] > 0 \tag{25}$$

The shear stress at the surface of the sheet is defined as

$$\tau_w = \frac{\mu_{nf}}{\rho_f u_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{1}{(1-\phi)^{2.5}}(Re_x)^{-\frac{1}{2}} f''(0) \tag{26}$$

where $Re_x = \frac{u_w x}{v_f}$ from (19) and (20) we obtain

$$-f''(0) = \frac{A^*(1 - R\alpha)}{\alpha} \tag{27}$$

4. Discussion

Fig. 1-3 have been made in order to see the effect of nanoparticle volume fraction (ϕ) and suction/injection parameters (R) on the velocity components respectively. It is found that for case the of suction in Fig. 1, the velocity components f' decreases with an increase in (R) and also the boundary layer thickness decreases. For the case of injection in Fig. 2, we have the opposite effect. Fig. 3 is drawn for the effects of nanoparticles volume fraction for both Ag- water and Cu- water on the velocity components f'. These figures show that f' decreases by increasing ϕ. The wall shear stress in Eq.(31) for different values of ϕ and (R) for both Ag- water and Cu- water are illustrated in Table 1-3. We present a comparison between the results obtained by present methods and Raftari and Yildrin [27] when $\phi = 0$ as shown 1n Table 1-2. Results show that the methods are in good agreement with one another, because in all the cases wall shear stress decreases when values of ϕ increases. Also in Table 3 wall shear stress increases when the values of (R) increases. Table 1 Comparison of the values of $f''(0)$ obtained by HPM, R-K and Raftari and Yildirim for the case of Cu-water.

	HPM	NM	Raftari and Yildirim[13]
ϕ	$f''(0)$	$f''(0)$	$f''(0)$
0.0	0.861187421	0.86119052388192	0.81506
0.1	0.985840673	0.98584139003671	-
0.2	1.015662492	1.01566299627196	-

Table 2 Comparison of the values of $f''(0)$ obtained by HPM, R-K and Raftari and Yildirim for the case of Ag-water.

	HPM	K-R	Raftari and Yildirim[13]
ϕ	$f''(0)$	$f''(0)$	$f''(0)$
0.0	0.861187421	0.86119052388192	0.81506
0.1	1.020461564	1.02046203933190	-
0.2	1.064171650	1.06417193184000	-

Table 3 $f''(0)$ for different values of R $\phi = 3$, for the case of Cu-water.

	HPM	K-R
$-R$	$f''(0)$	$f''(0)$
-0.3	0.9884334570	0.988434152232045
-0.2	1.0477589250	1.047759377782990
-0.1	1.1110892120	1.111089488157890
0.0	1.1784866240	1.178486780450180
0.1	1.2499722850	1.249972368659860
0.3	1.4050826750	1.405082692594350

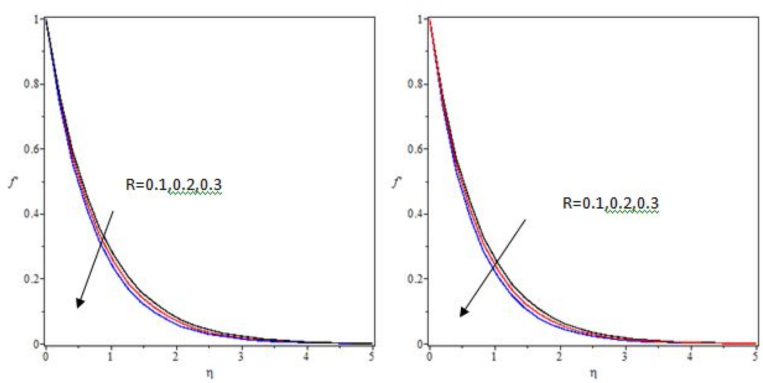

Figure 1: Velocity profile f' vs. η for $R > 0$ for the case. (a) Cu-water and (b) Ag-water

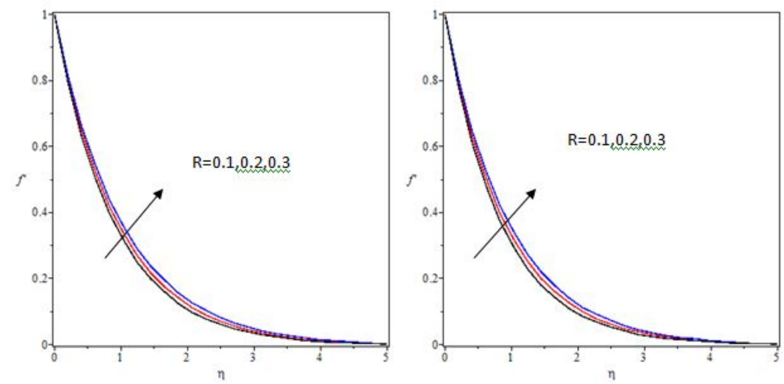

Figure 2: Velocity profile f' vs. η for $R < 0$ for the case. (a) Cu-water and (b) Ag-water

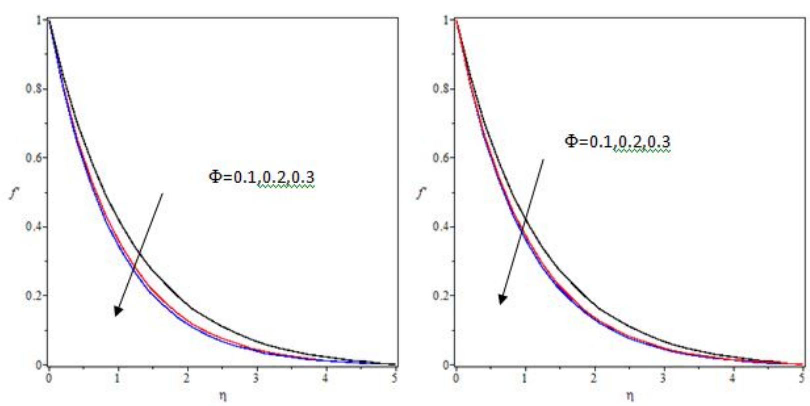

Figure 3: Velocity profile f' vs. η for different values of ϕ for the case. (a) Cu-water and (b) Ag-water

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