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Flow of an Incompressible MHD Third Grade Fluid Through a Cylindrical Pipe with Isothermal Wall and Joule Heating

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Abstract

In this paper, we considered the flow of an incompressible MHD third grade fluid through a cylindrical pipe with isothermal wall and Joule heating. Axial pressure-gradient was assumed to have induced the motion. The governing equations of the flow field are solved using perturbation method. Results are presented.

1. Introduction

With the emergence of polymer industry, petroleum industries and other types of pulp industries, in recent year, the non-Newtonian fluids have become very much important. Also with its complexity, it becomes difficult to suggest a single model which will exhibit all the properties of non-Newtonian fluids, as such various empirical and semi-emperical model have been put forward. Non-Newtonian fluids can be classified mainly into two groups such as differential type fluids and rate types fluid. Within these two groups, the differential type of fluids have received much attention from researchers. Example of such fluids in this group is the second grade fluid which is most acceptable because of its mathematical simplicity when compared with the third grade and the fourth fluids. Nargis Khan and Tahir Mahmood (2012) studied the influence of slip condition on the thin film flow of a third order fluid.

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To solve the coupled equations of the third grade fluid requires excessive computational effort and analytical solution which are almost impossible to obtain except strong simplification are made. Okoya —11— considered the disappearance of criticality for reactive third grade fluid with model viscosity in a flat channel [5]. Tahir Mahmood and Nargis Khan (2012) on thin film flow of a third grade fluid through porous medium over an inclined plane, employed the perturbation and homotopy perturbation methods and the results obtained from the two methods are in close agreement. Masoudi and Christie [10] analyzed the effects of temperature-dependent viscosity for the three separate cases treated by Szeri and Rajagopal [12].
According to Aiyesimi et al [3] on the analysis of unsteady MHD thin film flow of a third grade fluid with heat transfer down an inclined plane. They discovered that the variation of the velocity and temperature profile with the magnetic field parameter and the gravitational parameter depended on time. Aiyesimi et al [4] dealt with the effects of magnetic field on the MHD flow of a third grade fluid through inclined channel with Ohmic heating. They analyzed the couette flow, poiseuille and Couette-poiseuille flow and solved the resulting non- linear differential equation by employing regular perturbation technique.
Aiyesimi et al [1] studied MHD flow of a third grade fluid with heat transfer and slip boundary condition down an inclined plane. It was discovered that the result obtained by the traditional perturbation method was in close agreement with that obtained by homotopy perturbation technique. Aiyesimi et at [2] examined the viscous dissipation effect on the MHD flow of a third grade fluid down an inclined plane with mathematical formulation of the problem is established and solved in the next two sections respectively.

2.0 Mathematical Formulation

Following Makinde [9], we consider the steady flow of an incompressible MHD third grade fluid through a cylindrical pipe with isothermal wall. Axial pressure-gradient was assumed to have induced the fluid motion. For magnetohydrodynamically developed flow, both the velocity and the temperature fields depend on r only. Considering [5, 10, 14] and neglecting the reacting viscous fluid assumption, the governing equations for the momentum and energy balance can be represented as:

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)+\frac{\beta}{r}\frac{d}{dr}\left(r\left(\frac{du}{dr}\right)^3\right)-\sigma B_0^2 u=-\frac{dp}{dr} \tag{1}$$

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\left(\frac{du}{dr}\right)^2(\mu+\beta_3)\left(\frac{du}{dr}\right)^2+\gamma B_0 u^2=0 \tag{2}$$

Subject to the axial-symmetric and boundary conditions:

$$\frac{du}{dr}(0) = \frac{dT}{dr}(0) = 0, u(a) = 0, T(a) = 0 \tag{3}$$

where T is the temperature of the cylinder, u is the fluid velocity, T_0 is the plate temperature, μ is the coefficient of dynamic viscosity of the fluid, U is the characteristic fluid velocity, B_0 is the magnetic fluid effect, P is the pressure of the system, B_3 is the material coefficient and γ is the heat reaction.
Introducing the following non-dimensional variables into equations (1) - (3) for non-dimensionalizationn.

$$\theta = \frac{T}{T_0}, \bar{r} = \frac{r}{a}, u = \frac{u}{u_0} \tag{4}$$

and we obtain the following dimensionless momentum and energy equations:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) + \frac{\gamma}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)^3 - Mu = -1 \tag{5}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right) + B_r\left(\frac{du}{dr}\right)^2 + 2B_r\left(\frac{du}{dr}\right)^4 + Mu^2 = 0 \tag{6}$$

with

$$\frac{du}{dr}(0) = \frac{d\theta}{dr}(0) = 0, u(1) = 0, \theta(1) = 0 \tag{7}$$

where γ and M are the dimensionless non-Newtonian and viscous heating parameters respectively

2.0 Method of Solution

In order to solve equation (5), we introduce perturbation series as follows:

$$u = u_0 + = \gamma u_1 + 0(\gamma)^2 \text{ and } \theta = \theta_0 + \gamma\theta_1 + 0(\gamma)^2 (8)$$

with
$M = \gamma M$ and obtain the order γ^0 and order γ problems respectively as:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du_0}{dr}\right) = -1 \tag{9}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du_1}{dr}\right)+\frac{1}{r}\left(r\frac{du_0}{dr}\right)^3-Mu_0=0 \tag{10}$$

Then solving equations(9) and (10) using equation (7), we obtain:

$$u_0=\frac{1}{4}(1-r^4) \tag{11}$$

$$u_1=\frac{r^6}{48}+M\left(\frac{r^2}{16}-\frac{r^4}{64}\right)-\frac{1}{48}-\frac{3M}{64} \tag{12}$$

3.0 Heat Transfer Analysis

From equation (6), we obtain:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)+\gamma\left[\lambda\left(\frac{du}{dr}\right)^2 2B_r\left(\frac{du}{dr}\right)^4\right]+mu^2=0 \tag{13}$$

Using the series (8) in (13) and simplifying, we obtain

$$\gamma^0\colon\ \frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_0}{dr}\right)+B_r\left(\frac{du_0}{dr}\right)^2=0 \tag{14}$$

$$\gamma\colon\ \frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_1}{dr}\right)+2B_r\frac{du_0}{dr}\frac{du_1}{dr}+EcMu_0^2=0 \tag{15}$$

Using equation (11) in equation (14), we obtain:

$$\theta_0=B_r\left[\frac{1}{64}-\frac{r^4}{64}\right] \tag{16}$$

Using equations (11) and (12) in equation (15), we obtain:

$$\begin{aligned}\theta_1=&\frac{B_r r^8}{512}+\frac{B_rMr^4}{128}-\frac{B_rMr^4}{128}-\frac{EcMr^2}{64}+\frac{EcMr^4}{128}\\&-\frac{EcMr^6}{576}-\frac{B_r}{512}-\frac{B_rM}{128}+\frac{B_rM}{576}-\frac{EcM}{64}-\frac{EcM}{128}+\frac{EcM}{576}\end{aligned} \tag{17}$$

The solutions of the momentum and energy equations are respectively:

$$u(r)=\frac{1}{4}(1-r^2)+\gamma\left\{\frac{r^6}{48}+M\left(\frac{r^2}{16}-\frac{r^4}{64}\right)-\frac{1}{48}-\frac{3M}{64}\right\} \tag{18}$$

(19)

$$\theta(r) = B_r\left(\frac{1}{64} - \frac{r^4}{64}\right) + \gamma\left\{\frac{B_r r^8}{512} + \frac{B_r M r^4}{128} - \frac{B_r M r^4}{128} - \frac{EcMr^2}{64} + \frac{EcMr^4}{128}\right.$$
$$\left. - \frac{EcMr^6}{576} - \frac{B_r}{512} - \frac{B_r M}{128} + \frac{B_r M}{576} - \frac{EcM}{64} - \frac{EcM}{128} + \frac{EcM}{576}\right.$$

5.0 Results and Discussion

The temperature $\theta(r)$ and momentum $u(r)$ profiles were examined with various values of the physical paramters. Figures 1 and 2 shows that velocity increases with increase in magnetic parameter and γ respectively. While in figures 3,4,5 and 6 show that increase in γ, Brinkman number, M and the Eckerd number also increases the temperature profile.

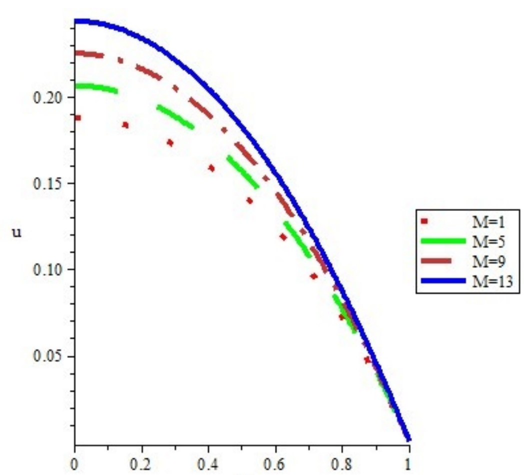

Fig 1: Graph of Velocity against Distance for various values of the Magnetic parameter

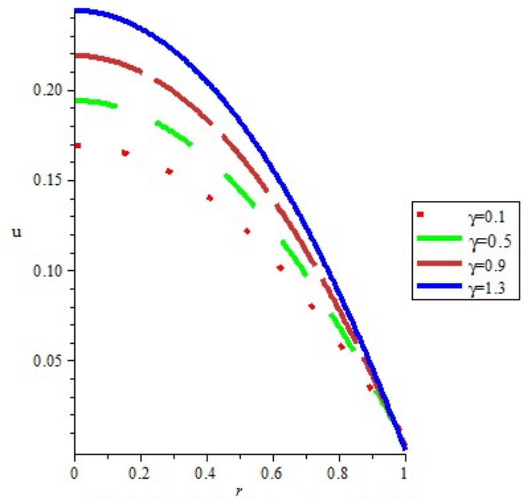

Fig.2: Velocity Profile for various values of γ

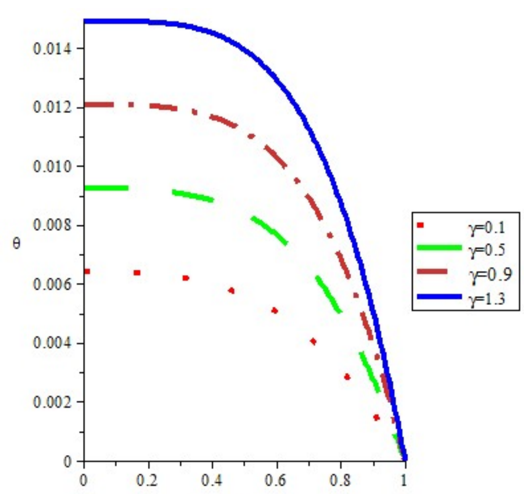

Fig.3: Temperature profile for values of γ

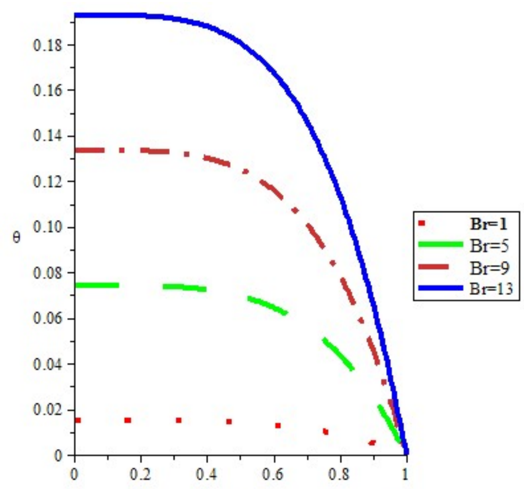

Fig.4: Temperature profile for values of Br

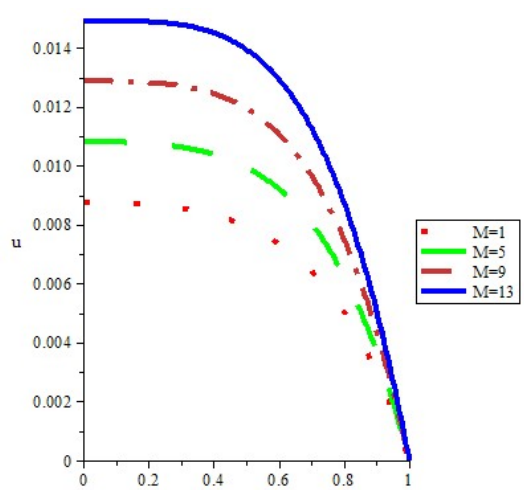

Fig 5 : Temperature profile for various values of M

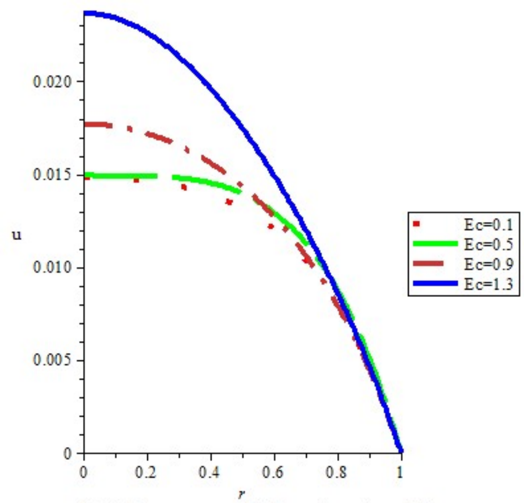

Fig.6: Temperature profile for various values of Ec

Conclusion

In this paper, we applied perturbation technique to examine the flow of an incompressible MHD third grade fluid through a cylindrical pipe with isothermal wall and Joule heating. The technique shows excellent agreement with Makindes result on thermal stability of a reactive third grade fluid with Hermite-Pade approximation technique.

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