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Adomian Decomposition Method (ADM) Solution of Boundary Layer Flow Past a Stretching Plate with Heat Transfer, Viscous Dissipations, and Grashof Number

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Abstract

The research work focuses on the solution of Magnetohydrodynamics (MHD) boundary layer flow past a stretching plate with heat transfer and viscous dissipation. The non-linear of momentum and energy equation are transformed into ordinary differential equation using similarity transformation, the resulting equations were solved using Adomian decomposition method (ADM). An attempt has been made to show the potentials and wide range application of the Adomian decomposition method in the comparison with the previous one in solving heat transfer problems. The Pade approximatants value ($\eta = 11[11, 11]$) was used on the difficulty at infinity. The results were compared by numerical technique method. A conclusion can be drawn from the results that ADM provides highly precise numerical solution for non-linear differential equations. The result were accurate especially for $\eta \leq 4$, a general equating terms of Eckert number (Ec), prandtl number (Pr), magnetic parameter $((\propto))$ and Grashof number were derived and were used to investigate velocity and temperature profiles in boundary layer.

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1. Introduction

The study of boundary layer over a stretching plate has generated much interest in recent years in view of its significant applications in industrial manufacturing process as glass fiber and paper production, hot rolling, wire drawing of plastic films, metal and polymer extraction and metal spinning. With these reasons, the study of flow and heat transfer in fluid has attracted the interest of many scientific investigators (researchers) in view of its application in engineering practices, particularly in chemical industries, such as the cases of boundary layer control, transpiration cooling and gaseous diffusion.

Tan (2008) studied a new branch of the temperature distribution of boundary layer flows over an impermeable stretching plate. Chen (1998) investigated the fluid flow and heat transfer on a stretching vertical sheet, and his work has been extended by Ishak et. al (2008) to hydro magnetic flow and they found that as the magnetic field increases, the surface skin friction as well as the surface Nusselt number decreases.

Anuj (2013) Examined Boundary layer flow past a stretching plate with heat transfer the nonlinear partial equations were transformed and the resulting ordinary differential equations were solved by using maple software. The effect of various parameters were presented and discussed. Also Aiyesimi (2004) studied a Convective MHD Flow of a Micropolar Fluid Past a Stretched Permeable Surface with heat generation or absorption was considered.

Hayat et.al (2009) investigated the hydro magnetic flow of a fluid bounded by a porous plate when the entire system about axis normal to the plate. And the result showed that the flow filed is appreciably influenced by the material parameter of the second grade fluid, notation, suction and blowing parameters.

Makinde et.al (2007) examined the free convention flow with thermal radiation and mass transfer past a moving vertical porous plate. The plate is maintained at a uniform temperature with uniform species concentration and the fluid is considered to be gray absorbing emitting. The coupled non linear momentum, energy and concentration equation governing the problem is obtained and made similar by introducing a time dependent length scale. The similarities Equation are then solved numerically by using superposition method.

Omowaye (2014) examined the steady arrhenius laminar free convective MHD flow and heat transfer past a vertical stretching sheet with viscous Dissipation an analysis of the effects of Arrhenius kinetics on hydro magnetic free concoctive flow of an electrically conducting fluid past a vertical stretching sheet kept at constant temperature with viscous dissipation is presented. A similarly transformation is used to reduce the governing partial differential equation into a system of ordinary differential equations which was solved numerically the effect of various parameters on the velocity and temperature profiles as well as the skin in graphs and tables. It was shown that the velocity and temperature increases as

local Ecket number (or various dissipation) increases.
To the best of our knowledge, the use of Adomain decomposition method to obtain the analytical solution to the two dimensional, steady incompressible boundary layer flows and heat transfer of a fluid with viscous dissipation has remained unexplored. The objective of the current work is to study the effect of various parameters that may occur on the velocity profile and temperature profile.

2. Problem formation

Consider two dimensional laminar boundary layer flows over a stretching plate in an incompressible electrically conducting fluid, where the $x-axis$ is along the stretching plate and $y-axis$ perpendicular to it, the applied magnetic field. B_0 is transversely to $x-axis$. Under the usual boundary layer approximations. It assumes that W and $\frac{\partial}{\partial z}$ are equal to zero. It also assumes that pressure gradient is neglected. The governed equations are continuity, momentum and energy under the influences of externally imposed transverse magnetic field are:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho}+gB(T-T_\infty) \tag{2}$$

Energy Equation

$$v\frac{\partial T}{\partial y}=\frac{v}{pr}\frac{\partial^2 T}{\partial y^2}+\frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

subject to boundary conditions

$$y=0\colon\, u=Ux, v=0, T=T_W;\;\, y=\infty, u=0, T=T_\infty \tag{4}$$

where v is the apparent kinematic viscosity, ρ_r is prandti number, σ is the electrical conductivity, u and v are the component of velocity along x and y directions, T is the temperature of the fluid in the boundary layer, T_∞ is the temperature of the fluid far away from the plate, T_W is the temperature of the plate, C_p is the specific heat capacity at constant pressure, B_0 is an external magnetic field, ρ is the fluid density, g is the acceleration due to gravity, B is the volumetric coefficient of thermal expansion.
By introducing the stream function φ

$$u=\frac{\partial \varphi}{\partial y} \tag{5}$$

(6) $$V = -\frac{\partial \varphi}{\partial x}$$

Equation (5-6) automatically satisfies the continuity equation (1) then the momentum equation (2) and energy equation (3) becomes:

(7)
$$\frac{\partial \varphi}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial \varphi}{\partial y}\right)\right) - \frac{\partial \varphi}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial \varphi}{\partial y}\right)\right) = \frac{\partial^2 \varphi}{\partial y^2}\left(\frac{\partial \varphi}{\partial y}\right) - \frac{\sigma \beta_0}{\rho}\left(\frac{\partial \varphi}{\partial y}\right) + gB(T - T_\infty)$$

(8) $$-\left(\frac{\partial \varphi}{\partial x}\left(\frac{\partial T}{\partial y}\right)\right) = \frac{\upsilon}{\rho_r}\left(\frac{\partial^2 T}{\partial y^2}\right) + \frac{\upsilon}{c_p}\left(\frac{\partial^2 \varphi}{\partial y^2}\right)^2$$

The dependent variable transformations are introduced as follows:

(9) $$\varphi = \sqrt[U]{\frac{(2-\beta)^{\upsilon}}{U}}\ x\ \frac{1}{2-\beta} f(\eta)$$

(10) $$U = U_{e(x)} f'_{(\eta)} = U\ x \frac{\beta}{2-\beta} f(\eta)$$

(11) $$\upsilon = -\sqrt[U]{\frac{\upsilon}{(2-\beta)U}}\ x \frac{\beta - 1}{2-\beta} + (f(\eta) + (\beta - 1)\eta(f'(\eta))$$

(12) $$T = (T_W - T_\infty)\ \theta\ (\eta)\ +\ T_\infty$$

(13) $$\eta = \sqrt[y]{(\frac{U}{2-\beta})\upsilon}\ x\ \frac{\beta - 1}{2-\beta}$$

where β is Falkner-scan pressure gradient parameter.

Substituting the expressions in (9-13) into (8) and (9), we have :

(14) $$f''' + ff'' - f'f' - \propto f' + G_r = 0$$

(15) $$\theta'' + \rho_r f\theta' + \rho_r Ec f''^2 = 0$$

where:

(16) $$\propto = \frac{\sigma \beta_0^2}{\rho U}, Ec = \frac{U_\ell^2}{c_p(T_W - T_\infty)} \text{ and } G_r = \frac{gB(T - T_\infty)}{U^2 x}$$

with corresponding boundary conditions:

(17) $$f'(0) = 1, f(0) = 0, \theta(0) = 1, f'(\infty) = 0, \theta(\infty) = 0$$

3. Adomain Decomposition Method

For the purpose of illustrating the method of adomain decomposition, we begin with the (deterministic) form $F(u) = g(t)$ where F is an ordinary differential operator with linear and nonlinear terms. We could represent the linear term Lu where L is the linear operator.
We write the linear term $Lu + Ru$ where we choose L as the highest-ordered derivative. Now L^{-1} is simply $n-$ fold integration for an n^{th} order. The remainder of the linear operator is R (in case where stochastic terms are present in linear operator, we can include a stochastic operator term Ru). The nonlinear term is represented by Nu. Thus, $Lu + Ru + Nu = g$ and we write $L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu$ for initial value problems we conveniently define $L^{-1} = \frac{d^n}{dt^n}$ as the n-fold definite integration operator from 0 to t. For the operator $L = \frac{d^2}{dt^2}$, for example we have,

$$L^{-1}Lu = u - u(0) - tu'(0)$$
$$U = u(0) + L^{-1}g - L^{-1}Ru - L^{-1}Nu$$

For the same operator equation, but now considering a boundary value problem, we let L^{-1} be an indefinite integral and write $u = A + Bt$ for the first two terms and evaluate A, B from the given condition the first three terms are identified as u_0 in the assumed decomposition

$$U = \sum_{n=0}^{\infty} U_n$$

Finally, assuming Nu is analytic, we write

$$Nu = \sum_{n=0}^{\infty} A_n(U_0, \ldots\ldots\ldots\ldots\ldots U_n)$$

where the A_n are specially generated (Adomain polynomials for specific nonlinearity).

4. Method of solution

The linear coupled differential equations (14) to (15) with boundary conditions (17) are solved using ADM methods. If ADM is applied on (14) to (16) and will define $L_1 = \frac{d^3}{d\eta^3}$ and $L_2 = \frac{d^2}{d\eta^2}$

(18) $$L_1 f = f'^2 + \propto f' - ff'' - G_r$$

(19) $$L_2\theta = -\rho_r f\theta' - \rho_r Ecf''^2$$

Applying inverse operator on equation (18) to (19), we have

(20) $$L_1^{-1}L_1 f = L^{-1}[f^{12} + \propto f' - ff'' - G_r]$$

$$L_2^{-1}L_2\theta = L_2^{-1}(-\rho_r f\theta') + L_2^{-1}(-\rho_r Ecf''^2) \tag{21}$$

From the boundary condition and taking $f''(0)=\beta, \theta'(0)=\lambda,$ where
$L_1^{-1} = \iiint(.)\ d(\eta)\ d(\eta)\ d(\eta)$and.
$L_2^{-1} = \iint(.)\ d(\eta)\ d(\eta)$
The ADM solution is obtained by:

$$\begin{aligned}\sum_{m=0}^{\infty} f_m(\tau) = +\frac{\eta^2\beta}{2} +\ L_1^{-1}\left(\sum_{m=0}^{\infty} Am\right) +\ L^{-1}\left(\sum_{m=0}^{\infty} \beta m\right)\\ +\ L^{-1}\left(\sum_{m=0}^{\infty} Cm\right) +\ L^{-1}\left(\sum_{m=0}^{\infty} Dm\right)\end{aligned} \tag{22}$$

$$\sum_{n=0}^{\infty} \theta_n(\eta) = 1 + \eta\lambda +\ L_2^{-1}\left(\sum_{n=0}^{\infty} E_n\right) +\ L_2^{-1}\left(\sum_{n=0}^{\infty} F_n\right) \tag{23}$$

where:

$$A_m = \sum_{v=0}^{n} f'_{m-v}f'_v \tag{24}$$

$$B_m = \sum_{m=0}^{n} f'_m \tag{25}$$

$$C_m = \sum_{v=0}^{n} f_{m-v}f''_v \tag{26}$$

$$D_m = \sum_{m=0}^{n} Gr_m \tag{27}$$

$$E_m = \sum_{v=0}^{n} f_{n-v}\theta' v \tag{28}$$

$$F_m = \sum_{v=0}^{n} f''_{n-v}f''_v \tag{29}$$

From (22) to (23), we have:

$$f_0(\eta) = \eta + \frac{\eta^2\beta}{2} \tag{30}$$

$$\theta_0(\eta) = 1 + \eta\lambda \tag{31}$$

For determination of the other component of $f(\eta)$ and $\theta(\eta)$, we have:
(32)
$$f_{m+1}(\eta) = L^{-1}(A_m) + L^{-1}(B_m) + L^{-1}(C_m) + L_2^{-1}(D_m), m = 0, 1, 2, \dots\dots\dots$$

(33) $$\theta_0(\eta) = L_2^{-1}(E_n) + L_2^{-1}(F_n), n = 0, 1, 2 \dots\dots$$

The general solutions are:

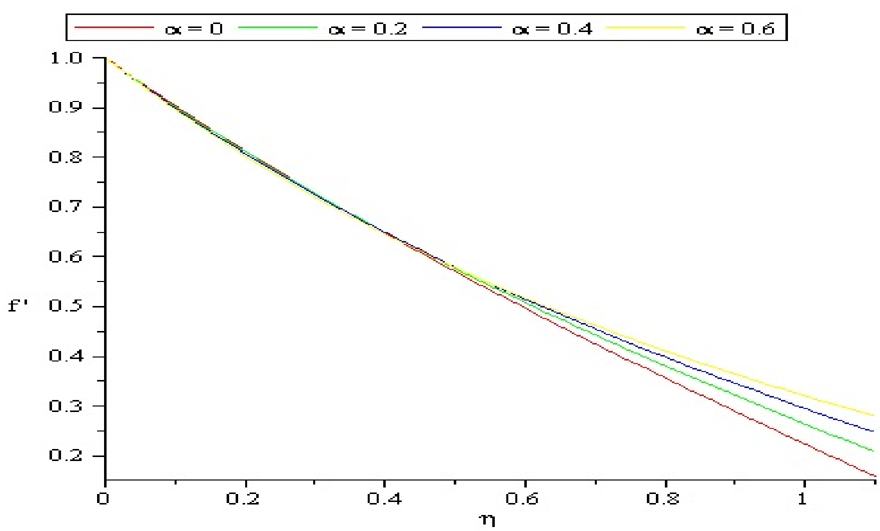
(34) $$f(\eta) = \sum_{m=0}^{\infty} = f_0 + f_1 + f_2 +$$

(35) $$\theta(\eta) = \sum_{n=0}^{\infty} \theta_n(\eta) = \theta_0 + \theta_1 + \theta_2$$

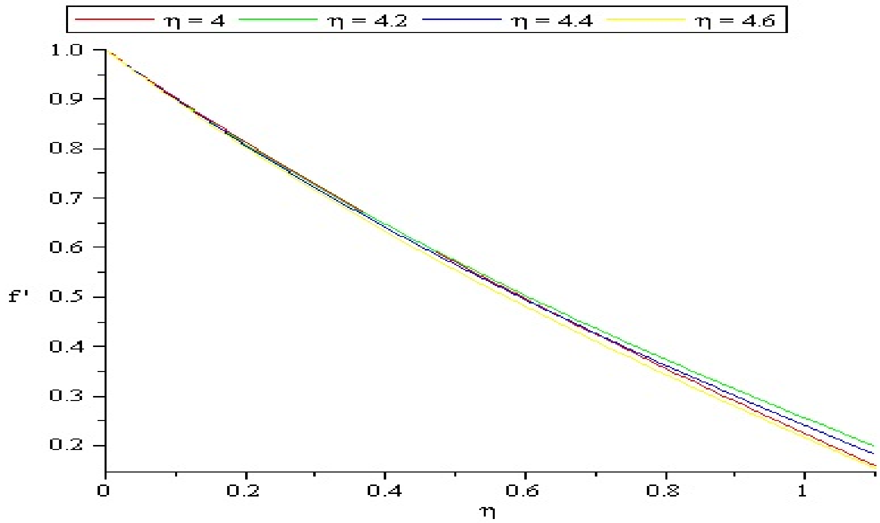
For conveniences, we use Maple-16 to compute the solutions

5. Results and Discussion

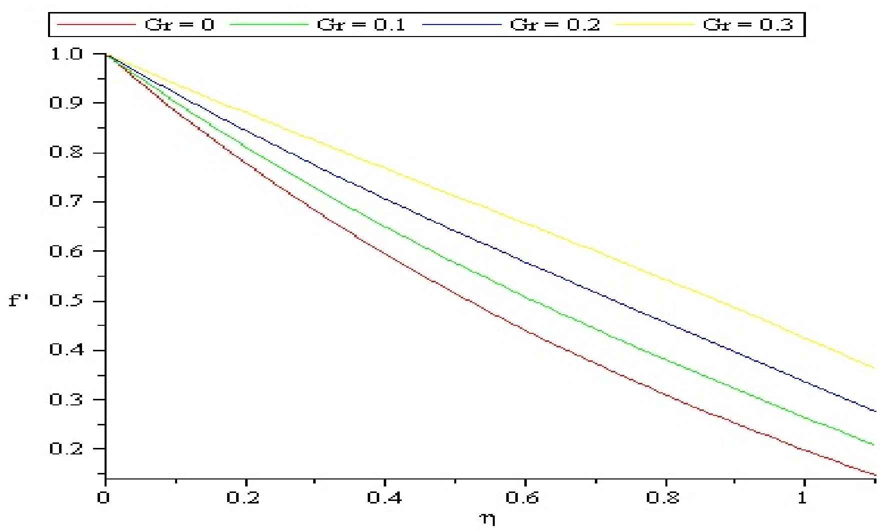
The system of nonlinear coupled ordinary differential equations (14) to (15) with boundary conditions (17) has been solved using the Adomain Decomposition Method. The effect of the physical parameters $\propto$, Ec, Pr and G_r on the velocity and temperature distribution which are shown in figures 1-5.



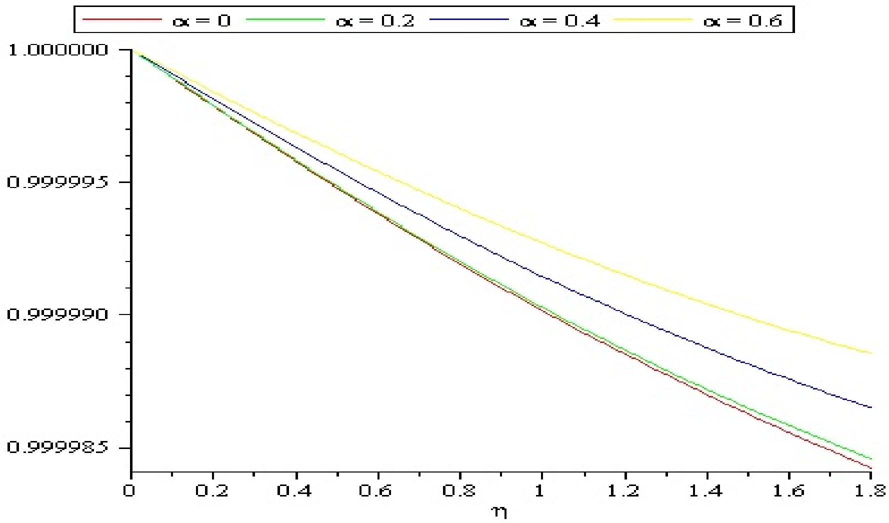
Graph of velocity profile $f'(\eta)$ against (η) for $\propto=$ 0, 0.2, 0.4 and 0.6 keeping $G_r =$ 0.1 and $\eta = 4$



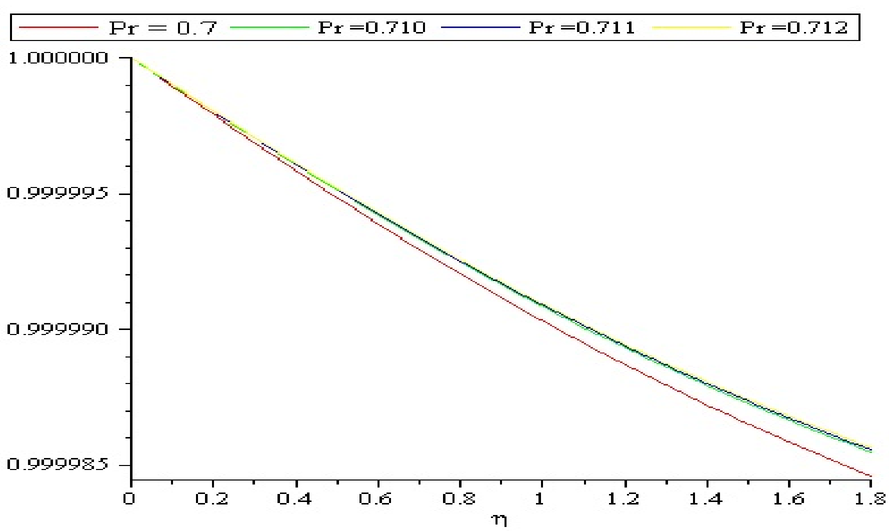
Graph of velocity profile $f'(\eta)$ against (η) for $\propto=$ 4, 4.2, 4.4 and 4.6 keeping $G_r =$ 0.1 and $\eta = 0.2$



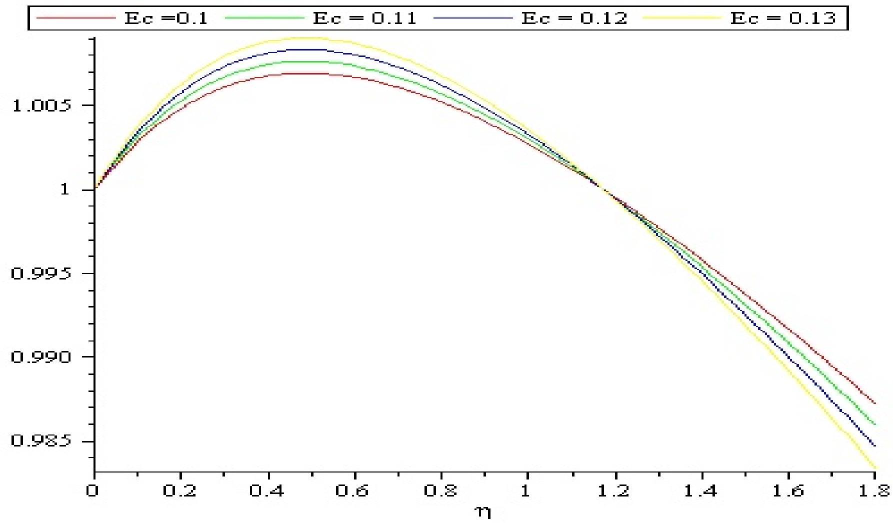
Graph of velocity profile $f'(\eta)$ against (η) for $\propto=$ 0.1, 0.2, 0.3 and 0.4 keeping $\eta =$ 4 and $\propto= 0.2$



Graph of temperature distribution $\theta(\eta)$ against the distance (η) for $\propto=\ 0,\ 0.2,\ 0.4$ and 0.6 keeping $Pr=\ 0.7,\ Ec=0$



Graph of temperature distribution $\theta(\eta)$ against the distance (η) for $Pr=\ 0.7,\ 0.710,\ 0.711$ and 0.712 keeping $\propto=\ 0.2,\ Ec=0$



Graph of temperature distribution $\theta(\eta)$ against the distance (η) for $Ec =$ 0.1, 0.11, 0.12 and 0.13 keeping $\propto=$ 0.2, $Pr = 0.7$

6. Discussion and Results

The local similarity equations governing the flow along with the boundary condition have been solved in the preceding section in order to give the detail of flow fields, thermal and velocity distributions. The effects of the main controlling parameters as they appear in the governing equation are discussed. Figure 1 to 3 show that velocity profiles increase for increased in Grashof numbers and magnetic parameter. Also velocity profile decreases for different value of η.Effect of prandti number parameters on the temperature and Eckert number parameters on the temperature profile is shown in figure 3 to 6, Temperature increases by slight increasing magnetic parameters and prandti number. It was shown that the temperature profile decreases as local Eckert number (or viscous dissipation parameter) increases.

7. Conclusion

The problem of convective MHD boundary layers flow past a stretching plate heat transfer, viscous dissipation and Grashof number with specified boundary conditions has been considered. A similarity transformation is used to reduce the governing partial differential equations, the resulting equation were solved using Adomain decomposition method (ADM). The results are presented to illustrate the detail of the flow condition and fluid properties.
It can also be concluded that:

(i) velocity profile increases with an increase in local Grashof number;
(ii) Increase in local Eckert number decreases temperature distribution;
(iii) The temperature decreases with an increase in the Eckert number.

References

[1] Aiyesimi, Y. M. (2004). *The Mathematical Analysis of Environmental Pollution of Freudich, non linear contaminant transport formulation.* Journal of Nigerian Association on of Mathematical physics, *8*, 8386.

[2] Anuj K. and Manoj K. (2013). *MHD Boundary layer flow past a stretching plate with heat transfer.* International Journal of Engineering and Sciences, *2*, 9-13.

[3] Chen C. H. (1998). *Laminar mixed convection adjacent to vertical, continuously stretching sheets.* Heat and Mass Transfer, *33*, 471 476.

[4] Hayet T., Husain Q. and Javed T. (2009). *The Modified Decomposition Method and pade approximant for the MHD flow over a non-linear stretching sheet, Non-linear Analysis.* Real Word Applications, *10* (2), 966-973.

[5] Ishak A., Nazar R. and Pop I. (2008). *Hydromagnetic flow and heat transfer adjacent to stretching vertical sheet.* Heat and Mass Transfer, *44*, 921-927.

[6] Makinde O. D., Olujuwon B. I., and Gbolagade A.W. (2007). *Adomain Decomposition approach to a Boundary layer flow with thermal radiation past a moving Vertical porous plate.* International Journal of Applied Mathematics and Mechanics, *3* (3), 62-70.

[7] Omowaye, A. J. (2014). *Steady Arrhenius laminar free convective MHD flow and heat transfer past a vertical stretching with viscous dissipation.* Journal of Nigeria Mathematics Society, *33*, 259-271.

[8] Tan (2008). *A New Branch of the Temperature Distribution of Boundary Layer Flows over an Impermeable Stretching Plate.* Heat and Mass Transfer, *44*, 501 - 504.