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MATHEMATICAL DYNAMICS OF THE INTERACTION OF 2-3 PREY-PREDATOR NON-DIFFUSIVE ECOLOGICAL SYSTEM

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Abstract

A diagram illustrating the relationship between three (3) competing predators who also compete amongst themselves and two independent Prey is shown to see the nature of interdependence among the species. To this end, it is considered that the free population (without interaction) is the solution to the Logistic equation of individual specie. A semi analytical approach is developed for solving the dynamics of a 2-3 Prey-Predator system in a non diffusive state and a diffusive state. The resultant solution is then analysed to see the impact of predation on the population of both the Prey and Predators. It was observed that the growth rate in the population of the Prey is inversely proportional to the growth rate of population of the Predators. The equilibrium state of the system is also examined.

1. Introduction

The question of two predator species competing for one prey was brought to the fore by ecological literature written by May (1976), Hsu (1981), Harrison

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(1979), Gopalsamy (1986). A mathematical model for the two predator species exploiting a single prey was proposed by Hsu(1981). He found out that when the inter-specific interference coefficient is small, the winner competes with rivals successfully. Mitra et al. (1992) studied the permanent coexistence and global stability of a simple Lotka-Volterra type mathematical model of a living resource supporting two predators. They showed that the permanent coexistence of the system depends on the threshold of the ratio between the coefficients of numerical responses of the two predators/consumers. Dubey Das (2000) proposed a Guass-type model with diffusion of which is analyzed. For the research, they considered a system of two predators competing with interference for a limited prey. They showed that in the absence of intra-specific interaction of the predator series, the interior equilibrium is unstable.

For this research work, we propose a model of three interacting predators competing against two preys for which both preys are independent and are not competing against each other. To achieve this, a competitive Lotka-Volterra equation is employed. To ensure for proper analysis, some semi- analytic method of solution shall be employed.

2. Mathematical formulation of problem

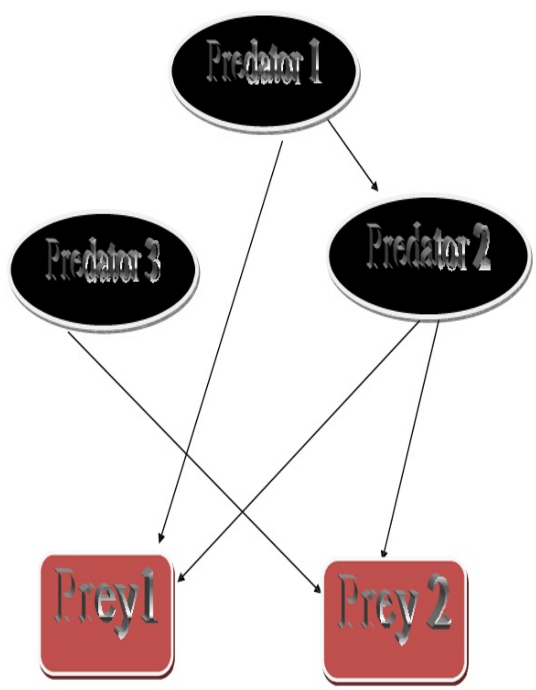


Diagram showing the 2-3 Prey-predator Interactions we can deduce the following:

(1) $$\frac{dx_1}{dt} = x_1(r_1 - \alpha_1 y_1 - \alpha_2 y_2)$$

(2) $$\frac{dx_2}{dt} = x_2(r_2 - \beta_1 y_2 - \beta_2 y_3)$$

(3) $$\frac{dy_1}{dt} = -y_1(s_3 + u_3 - r_3 - \sigma_1 x_1 - \sigma_2 y_2 - \sigma_3 y_3)$$

(4) $$\frac{dy_2}{dt} = -y_2(s_4 + u_4 - r_4 - \delta_1 x_1 - \delta_2 x_2 + \delta_3 y_1)$$

(5) $$\frac{dy_3}{dt} = -y_3(s_5 + u_5 - r_3 - \phi_1 x_2 + \phi_2 y_1)$$

Initial Conditions
$x_1(0) = x_{10}$ $x_2(0) = x_{20}$ $y_1(0) = y_{10}$ $y_2(0) = y_{20}$ $y_3(0) = y_{30}$
y_1 is the 1^{st} Predator
y_1 is the 2^{nd} Predator
y_1 is the 3^{rd} Predator
x_1 is the 1^{st} Prey
x_2 is the 2^{nd} Prey
${}_1, \alpha_2, \beta_1, \beta_2, \sigma_1, \sigma_2, \sigma_3, \delta_1, \delta_2, \delta_3, \phi_1$ and ϕ_2 are interaction coefficients.
r_1 and r_2 are birth rates of 1^{st} Prey and 2^{nd} Prey respectively
s_3, s_4 and s_5 are death rate by old age of 1^{st} Predator, 2^{nd} Predator and 3^{rd} Predator respectively.
u_3, u_4 and u_5 are death rate by accident of 1^{st} Predator, 2^{nd} Predator and 3^{rd} Predator respectively.
t is the time.

3.0 Solution to Resulting Models
To solve the models, we make use of the equation below:

(6) $$p_n = \frac{p_0 e^{at}}{(1 - \alpha.p_0(1 - e^{(at)}))} - \int_0^t \frac{e^{a(t-s)}(1 - \alpha._0(1 - e^{as})}{(1 - \alpha.p_0(1 - e^{at}))} F ds$$

For Prey 1:
We assume free population (without interactions) of Prey 1 to be:

(7) $$\frac{dx_1}{dt} = r_1 x_1(1 - \Omega_1 x_1)$$

Based on Volterra (1927),
Considering (3.1), we insert the term $r_1\Omega_1 x_1^2$

$$\begin{aligned} \frac{dx_1}{dt} x_1(r_1 - r_1\Omega_1 x_1 + r_1\Omega_1 x_1 - \alpha_1 y_1 - \alpha_2 y_2) \\ \frac{dx_1}{dt} r_1 x_1(1-\Omega_1) + x_1(r_1\Omega_1 x_1 - \alpha_1 y_1 - \alpha_2 y_2) \end{aligned} \tag{8}$$

where
$p = x_1$
$a = r_1$
$\alpha = \Omega_1$
Solving (8) using (6), we have

$$x_1 = \frac{x_{10}e^{r_1 t}}{(1-\Omega_1 x_{10}(1-e^{r_1 t})} \left\{ 1 - \left\{ \begin{array}{l} \frac{\alpha_1}{\alpha_3\Omega_3} \ln(1-\Omega_3 y_{10}(1-e^{a_3 t})) \\ +\frac{\alpha_2}{a_4\Omega_4} \ln(1-\Omega_4 y_{20}(1-e^{a_4 t})) \\ -\ln(1-\Omega_1 x_{10}(1-e^{r_1 t})) \end{array} \right. \right\} \tag{9}$$

Likewise for Prey 2, Predator 1, Predator 2 and Predator 3, we have the following results:

$$x_2 = \frac{x_{20}e^{r_2 t}}{(1-\Omega_2 x_{20}(1-e^{r_2 t})} \left\{ 1 - \left\{ \begin{array}{l} \frac{\beta_1}{\alpha_4\Omega_4} \ln(1-\Omega_4 y_{20}(1-e^{a_4 t})) \\ +\frac{\beta_2}{a_5\Omega_5} \ln(1-\Omega_5 y_{30}(1-e^{a_5 t})) \\ -\ln(1-\Omega_2 x_{20}(1-e^{r_2 t})) \end{array} \right. \right\} \tag{10}$$

$$y_1 = \frac{y_{10}e^{a_3 t}}{(1-\Omega_3.y_{10}(1-e^{a_3 t})} \left\{ 1 + \left\{ \begin{array}{l} \frac{\sigma_1}{r_1\Omega_1} \ln(1-\Omega_1 x_{10}(1-e^{r_1 t})) \\ +\frac{\sigma_2}{a_4\Omega_4} \ln(1-\Omega_4 y_{20}(1-e^{a_4 t})) \\ +\frac{\sigma_3}{a_5\Omega_5} \ln(1-\Omega_5 y_{30}(1-e^{a_5 t})) \\ +\ln(1-\Omega_3 y_{10}(1-e^{a_3 t})) \end{array} \right. \right\} \tag{11}$$

$$y_2 = \frac{y_{20}e^{a_4 t}}{(1-\Omega_4.y_{20}(1-e^{a_4 t})} \left\{ 1 - \left\{ \begin{array}{l} \frac{-\delta_1}{r_1\Omega_1} \ln(1-\Omega_1 x_{10}(1-e^{r_1 t})) \\ -\frac{\delta_2}{r_2\Omega_2} \ln(1-\Omega_2 x_{20}(1-e^{r_2 t})) \\ +\frac{\delta_3}{a_3\Omega_3} \ln(1-\Omega_3 y_{10}(1-e^{a_3 t})) \\ -\ln(1-\Omega_4 y_{20}(1-e^{a_4 t})) \end{array} \right. \right\} \tag{12}$$

$$y_3 = \frac{y_{30}e^{a_5 t}}{(1-\Omega_5.y_{30}(1-e^{a_5 t})} \left\{ 1 - \left\{ \begin{array}{l} \frac{-\phi_1}{r_2\Omega_2} \ln(1-\Omega_2 x_{20}(1-e^{r_2 t})) \\ +\frac{\phi_2}{a_3\Omega_3} \ln(1-\Omega_3 y_{10}(1-e^{a_3 t})) \\ -\ln(1-\Omega_5 y_{30}(1-e^{a_5 t})) \end{array} \right. \right\} \tag{13}$$

Where
$a_3 = (r_3 - s_3 - u_3)$
$a_4 = (r_4 - s_4 - u_4)$

$a_5 = (r_5 - s_5 - u_5)$

4.0 Equilibrium state of the Model

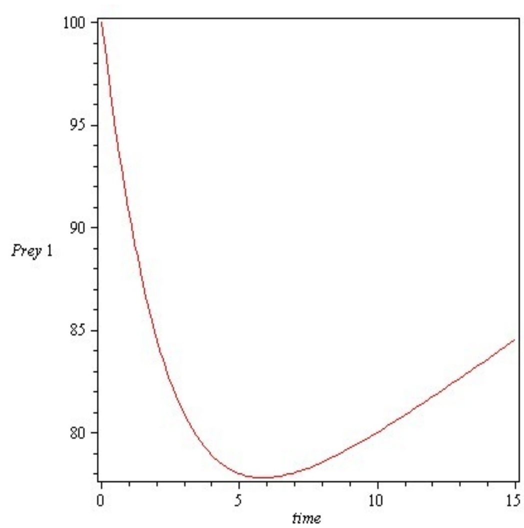
The equilibrium positions of the prey-predator interaction dynamics is obtained as thus:

$$\frac{dx_1}{dt} = \frac{dx_2}{dt} = \frac{dy_1}{dt} = \frac{dy_2}{dt} = \frac{dy_3}{dt} = 0. \tag{14}$$

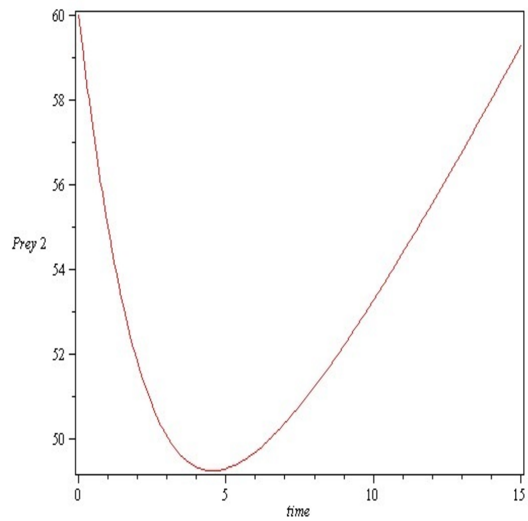
Therefore, there exist the following equilibria namely $E_0(0,0,0,0,0)$, $E_1(0,x_2,y_1,y_2,y_3)$, $E_2(x_1,0,y_1,y_2,y_3)$, $E_3(x_1,x_2,0,y_2,y_3)$, $E_4(x_1,x_2,y_1,0,y_3)$, $E_5(x_1,x_2,y_1,y_2,0)$ and $E_6(x_1,x_2,y_1,y_2,y_3)$.

Numerical Applications

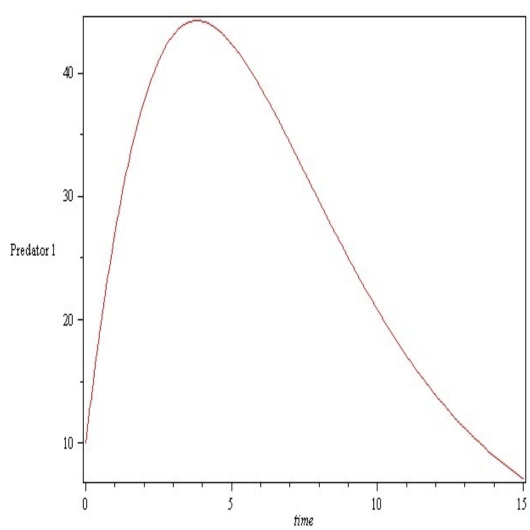
Equation (9)-(13) which is the solution to the model is assigned feasible values for its interaction rates to see its dynamics over time.



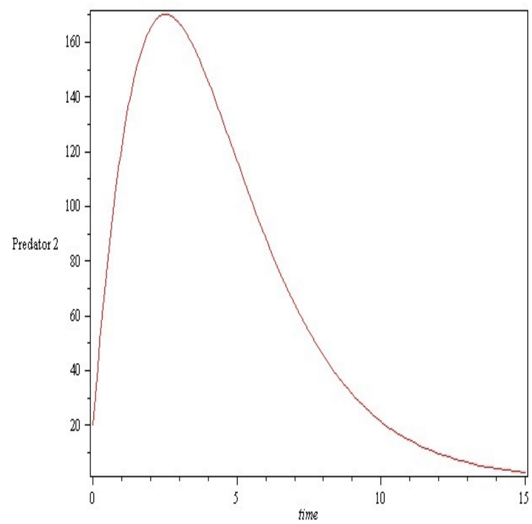
Initial Prey 1 Graph



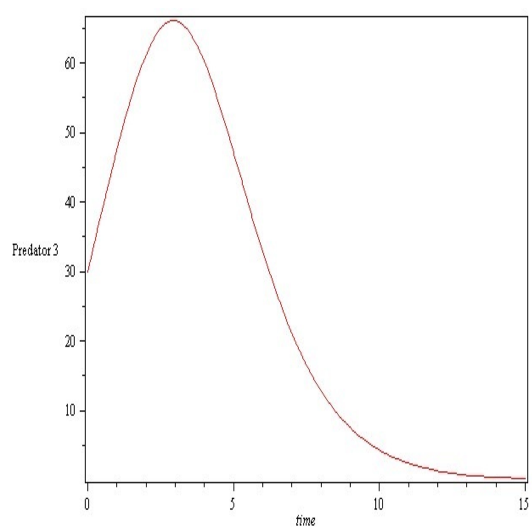
Initial Prey 2 Graph



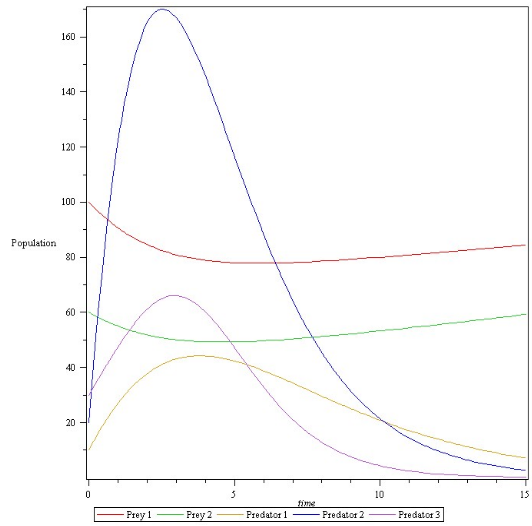
Initial Predator 1 Graph



Initial Predator 2 Graph



Initial Predator 3 Graph



Initial 2-3 Prey-Predator Graph

Fig 1 and Fig 2 shows that with direct effect of predation, the population of Prey 1 decreases until there is limited prey for the predators. Fig 3,4 and 5 shows that there is an increase in the population of the Predators until it gets to a threshold then it begins to decrease. The above shows that the increase in populations of the Predator, the prey population decreases reaching a point where it starts to increase of which also the Predator population begins to decrease.

5. Conclusion

Multiple Prey-Predator interactions often exhibit similar growth pattern among the prey and also among the predators. At equilibria points often the absence of one or two specie leads the stagnation of the population of other species. Hence, we note that the method of solution aided in solving the resulting modelling equation of the Prey-Predator system by giving us a solution for each of the species.

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