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Modeling and Analytical Simulation of a Laminar Premixed Flame Impinging on a Normal Solid Surface

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Abstract

The two most relevant applications of laminar, compressible boundary layer theory are the flat plate flow and stagnation point flow. In this paper, an analytical method for studying chemically reacting flow in laminar premixed flame of carbon monoxide/oxygen mixture in the region of the stagnation point in the presence of work due to compression and heat generated due to viscous dissipation is presented. The governing non-linear partial differential equations describing the phenomenon have been transformed by a similarity transformation into a system of ordinary differential equations, which are solved analytically using iteration perturbation method together with direct integration. The results obtained show that velocity increases as Prandth number increases. Also, Biot number decreases the fluid velocity and enhances the species concentration and flame temperature.

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1. Introduction

One of the most important pollutants in combustion phenomena is carbon monoxide which is a toxic component of air. Catalytic oxidation of carbon monoxide to carbon dioxide at ambient temperature and pressure is an important process for respiratory protection. In particular, the process is widely adopted by mining industries and has also found applications in deep-sea diving, space exploration and carbon dioxide lasers [3].

The two most relevant applications of laminar, compressible boundary layer theory are the flat plate flow and stagnation point flow. Both cases give insight into the general effect of compressibility on boundary layer flows and are very useful for estimating the friction and heat transfer on slender bodies or blunt bodies, respectively [3].

Zeldovitch et al. [9] were the first to model purely gaseous premixed flames with a chain mechanism using a two-step chemical mechanism. Other work followed using this non-linear mechanism (see, for example, a modified mechanism by Dold et al. [2]). Olayiwola et al. [6] considered a steady, adiabatic, premixed laminar flame in an approximation in which all species concentrations can be related to the temperature T as the single dependent variable. An analytical solution for the model with variable thermal conductivity was provided. In a more recent paper, Olayiwola et al. [7] studied mathematically the chemical kinetics of a laminar premixed flame. They examined the properties of solution. To simulate the flow, they assumed that the incoming mixture is at the burner temperature.

Several works have been done on an impinging laminar flame jets. Li et al. [5] showed that there exist two different solutions for the flow field in some range of geometric and flow parameters. Sibulkin [8] derived a semi-analytical relation for laminar heat transfer of impinging flow to a body of revolution. Hammoud and Souidi [3] presented a numerical method for the study of chemically reacting flow in laminar premixed flame of carbon monoxide/oxygen mixture in the region of the stagnation point. To simulate the flow, they neglected the effects of both thermal and viscous dissipation.

The objective of this paper is to obtain an analytical solution for prediction of fluid velocity, flame temperature and species concentration distribution of a laminar premixed flame impinges on a plane solid surface in the presence of work due to compression and heat generated due to viscous dissipation. As in [3], we assume constant Prandtl and Schmith numbers and specific heat at constant pressure. To simulate the flow analytically, it is further assumed that there is no mass or heat transfer at the surface.

2. Model Formulation

The system of governing equations to be solved for a two-dimensional, steady, compressible, laminar boundary layer without body forces and bulk heat transfer is as follows:

Conservation of mass

$$\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0 \tag{1}$$

Conservation of momentum

$$\left.\begin{array}{c} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \\ \frac{\partial p}{\partial y} = 0 \end{array}\right\} \tag{2}$$

Conservation of species

$$\left(u\frac{\partial Y_k}{\partial x} + v\frac{\partial Y_k}{\partial y}\right) = \frac{1}{Sc}\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial Y_k}{\partial y}\right) + \frac{1}{\rho}\omega M_k \tag{3}$$

Conservation of energy

$$\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{u}{\rho c_p}\frac{\partial p}{\partial x} + \frac{1}{\Pr}\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial T}{\partial y}\right) + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{\rho c_p}\sum_{k=1}^{N}\omega M_k h_k \tag{4}$$

Equation of State

$$p = \rho RT \tag{5}$$

Boundary Conditions

The boundary conditions at the surface, i.e., $y = 0$ are given by the no-slip velocity condition without mass transfer or heat transfer. Since $u(y = 0) = 0$, energy transfer to / from fluid occurs by conduction only. Thus

(6)

$$u(x,0) = 0, \quad v(x,0) = 0, \quad \left.\frac{\partial Y_k}{\partial y}\right|_{y=0} = 0, \quad -\lambda_f\left.\frac{\partial T}{\partial y}\right|_{y=0} = h(T_w - T_e), \quad T_w \neq T_e$$

At the edge of the boundary layer, the viscous flow inside the boundary layer is required to smoothly transition into the inviscid flow outside the boundary layer. Then

$$u(y\to\infty) \to U_e(x), \quad Y_k(y\to\infty) \to Y_e(x), \quad T(y\to\infty) \to T_e(x), \tag{7}$$

where the subscripts w and e represents condition at the wall and edge of the boundary layer respectively, T is the flame temperature, Y_k is the mass fraction of species k, ρ is the density, x is the direction along the surface creating the boundary layer, y is the direction normal to the surface, u is velocity in the x direction, v is velocity in the y direction, λ is the thermal conductivity, λ_f is the fluid thermal conductivity, h is the convective heat transfer coefficient, μ is the viscosity, p is the pressure, R is the gas constant, c_p is the specific heat at constant pressure, h_k is the enthalpy formation of species k, D is the diffusion coefficients and ω is the rate term in Arrhenius form given as

$$\omega = \upsilon_f A\rho^{\upsilon_f+\upsilon_o} M_o^{-\upsilon_o} M_f^{-\upsilon_f} Y_o^{\upsilon_o} Y_f^{\upsilon_f}\, e^{-\frac{E}{RT}}, \tag{8}$$

where $Y_f,\ Y_o$ are the mass fraction of the fuel and oxidizer, and $M_f,\ M_o$ are molecular weights. $\upsilon_f,\ \upsilon_o$ are correspond stoichiometric coefficients, E is the activation energy and A is the pre exponential constant.
We make additional assumption that c_p is a constant and equal for all species. Although these assumptions could be relaxed in the future, they considerably simplify the equations. Then (3) and (4) can be simplified as

$$\left(u\frac{\partial Y}{\partial x}+v\frac{\partial Y}{\partial y}\right)=\frac{1}{Sc}\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial Y}{\partial y}\right)+\frac{1}{\rho}\omega M \tag{9}$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{u}{\rho c_p}\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Pr}}\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial T}{\partial y}\right)+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\omega M\Delta h \tag{10}$$

The pressure gradient term in energy equation (10), can be eliminated by multiplying the momentum equation (2) by $\frac{u}{c_p}$ and adding the result to the energy equation (10). This results in:

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)+\frac{u}{c_p}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{u}{\rho c_p}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\frac{1}{\mathrm{Pr}}\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial T}{\partial y}\right)+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\omega M\Delta h \tag{11}$$

3. Method of Solution

3.1 Variable Transformation

For the similarity transformations and the corresponding similar solutions, the compressible stream function can be defined by:

$$\frac{\partial\psi}{\partial y}=\rho u \tag{12}$$

$$\frac{\partial\psi}{\partial x}=-\rho v \tag{13}$$

Equation (12) and (13) automatically satisfy the continuity equation (1). Then, the momentum equation (2), species equation (9) and energy equation (11) become:

$$\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial y}\right)-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial y}\right)\right) \tag{14}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial Y}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial Y}{\partial y}=\frac{1}{Sc}\frac{\partial}{\partial y}\left(\mu\frac{\partial Y}{\partial y}\right)+\omega M \tag{15}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}+\frac{1}{\rho c_p}\frac{\partial\psi}{\partial y}\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial y}\right)-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial y}\right)\right)=\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial y}\left(\mu\frac{\partial T}{\partial y}\right)+\frac{1}{\rho c_p}\frac{\partial\psi}{\partial y}\frac{\partial}{\partial y}\left(\mu\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial y}\right)\right)+\frac{\mu}{c_p}\left(\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial y}\right)\right)^2-\frac{1}{c_p}\omega M\Delta h \tag{16}$$

The dependent variable transformations are introduced as follows:

$$\psi\left(x,y\right)=\rho U\sqrt{\frac{\left(2-\beta\right)\upsilon}{U}}x^{\frac{1}{2-\beta}}f\left(\eta\right) \tag{17}$$

$$u\left(x,y\right)=U_{e}\left(x\right)f'\left(\eta\right)=Ux^{\frac{\beta}{2-\beta}}f'\left(\eta\right) \tag{18}$$

$$v\left(x,y\right)=-U\sqrt{\frac{\upsilon}{\left(2-\beta\right)U}}x^{\frac{\beta-1}{2-\beta}}\left(f\left(\eta\right)+\left(\beta-1\right)f'\left(\eta\right)\right) \tag{19}$$

$$Y\left(x,y\right)=Y\left(\eta\right) \tag{20}$$

$$T\left(x,y\right)=T\left(\eta\right) \tag{21}$$

Independent variable transformation is introduced as follows:

$$\eta=y\sqrt{\frac{U}{\left(2-\beta\right)\upsilon}}x^{\frac{\beta-1}{2-\beta}}, \tag{22}$$

where β is Falkner-Skan pressure gradient parameter.
Introducing equations (17), (18), (19), (20), (21) and (22) into the momentum equation (14), species equation (15) and energy equation (16) and using Euler's equation at the edge of the boundary layer, i.e., $\frac{\partial p}{\partial x}=-\rho U_{e}\frac{dU_{e}}{dx}$, results in:

$$\frac{\beta}{2-\beta}f'^{2}+\frac{\beta-1}{2-\beta}\eta f'f''-\frac{1}{2-\beta}\left(ff''+\left(\beta-1\right)\eta f'f''\right)=\frac{\beta}{2-\beta}+\frac{1}{\rho\left(2-\beta\right)\upsilon}\left(\mu f'''+\mu'f''\right) \tag{23}$$

$$\begin{gathered}\tfrac{\beta-1}{2-\beta}\eta f'Y'-\tfrac{1}{2-\beta}\left(f+\left(\beta-1\right)\eta f'\right)Y'=\tfrac{1}{Sc}\tfrac{1}{\rho(2-\beta)\upsilon}\left(\mu Y''+\mu'Y'\right)+\\ \frac{\upsilon_{f}A\rho^{\upsilon_{f}+\upsilon_{0}-1}M^{1-\left(\upsilon_{f}+\upsilon_{0}\right)}Y^{\upsilon_{f}+\upsilon_{0}}e^{-\frac{E}{RT}}}{Ux^{\frac{2(\beta-1)}{2-\beta}}}\end{gathered} \tag{24}$$

$$\begin{gathered}\tfrac{\beta-1}{2-\beta}\eta f'T'-\tfrac{1}{2-\beta}\left(f+\left(\beta-1\right)\eta f'\right)T'+\tfrac{U_{e}^{2}}{c_{p}}\left(\tfrac{\beta}{2-\beta}f'^{3}+\tfrac{1}{2-\beta}ff'f''+\tfrac{2(\beta-1)}{2-\beta}\eta f'^{2}f''\right)\\ =\tfrac{1}{\Pr}\tfrac{1}{\rho(2-\beta)\upsilon}\left(\mu T''+\mu'T'\right)+\tfrac{U_{e}^{2}}{\rho c_{p}(2-\beta)\upsilon}\left(\mu f'f'''+\mu'f'f''\right)+\tfrac{U_{e}^{2}\mu}{\rho c_{p}(2-\beta)\upsilon}f''^{2}-\\ \frac{\Delta h\upsilon_{f}A\rho^{\upsilon_{f}+\upsilon_{0}-1}M^{1-\left(\upsilon_{f}+\upsilon_{0}\right)}Y^{\upsilon_{f}+\upsilon_{0}}e^{-\frac{E}{RT}}}{c_{p}Ux^{\frac{2(\beta-1)}{2-\beta}}}\end{gathered} \tag{25}$$

From an equation of state, the density is a function of temperature and pressure, i.e., $\rho=\rho\left(T,P\right)$. However, the pressure is assumed constant across the boundary layer. Therefore, the density and viscosity can be assumed to be a function of temperature only, i.e., $\rho=\rho\left(T\right)$, $\mu=\mu\left(T\right)$. Introducing a Chapman-Rubesin viscosity law, with $w = 1$ and using the conditions at the edge of the boundary layer as reference, results in:

$$\frac{\mu}{\mu_e} = c\frac{T}{T_e}, \quad (w = 1) \tag{26}$$

from which, we have
$\mu = \frac{c\mu_e T}{T_e}$ and $\mu' = \frac{c\mu_e T'}{T_e}$, (**??**)
where c is a constant.
In the above analysis, we consider an important special case, $\beta = 1$, corresponding with stagnation point flow and by using (**??**) in equations (23), (24) and (25) and introducing the dimensionless temperature and species mass fraction as:

$$\theta = \frac{E}{RT_e^2}\left(T - T_e\right), \quad \varphi = \frac{Y}{Y_e} \tag{27}$$

results in:

$$f''' + ff'' + 1 - f'^2 + \frac{\in}{(1+\in\theta)}f''\theta' = 0 \tag{28}$$

$$\frac{1}{Sc}\varphi'' + f\varphi' + \frac{1}{Sc}\frac{\in}{(1+\in\theta)}\varphi'\theta' + \sigma\varphi^{\upsilon_f+\upsilon_0}e^{\frac{\theta}{1+\in\theta}} = 0 \tag{29}$$

$$\begin{aligned}&\tfrac{1}{\Pr}\theta'' + +f\theta' + \tfrac{1}{\Pr}\tfrac{\in}{(1+\in\theta)}\theta'^2 - \tfrac{Ec}{\in}\left(f'^3 + ff'f'' - f'f''' - f''^2\right) + \\ &\tfrac{Ec}{(1+\in\theta)}f'f''\theta' - \delta\varphi^{\upsilon_f+\upsilon_0}e^{\frac{\theta}{1+\in\theta}} = 0\end{aligned} \tag{30}$$

together with boundary conditions

$$\left.\begin{aligned} f(0) = f'(0) = 0, &\quad f'(\eta\to\infty) = 1 \\ \varphi'(0) = 0, &\quad \varphi(\eta\to\infty) = 1 \\ \theta'(0) = \tfrac{Bi}{\in}(1-\alpha), &\quad \theta(\eta\to\infty) = 0 \end{aligned}\right\} \tag{31}$$

where
$\Pr = \frac{\mu c_p}{\lambda}$: Prandtl number, $Sc = \frac{\mu}{\rho D}$: Schmidt number, $Ec = \frac{U_e^2}{c_p T_e}$: Eckert number, $\in = \frac{RT_e}{E}$: Dimensionless Activation Energy, $\sigma = \frac{\upsilon_f A\rho^{\upsilon_f+\upsilon_0-1}M^{1-\left(\upsilon_f+\upsilon_0\right)}Y_e^{\upsilon_f+\upsilon_0-1}e^{-\frac{E}{RT_e}}}{U}$,
$\delta = \frac{\Delta h\upsilon_f A\rho^{\upsilon_f+\upsilon_0-1}M^{1-\left(\upsilon_f+\upsilon_0\right)}Y_e^{\upsilon_f+\upsilon_0}e^{-\frac{E}{RT_e}}}{c_p\in T_e U}$: Frank-Kamenetskii number, $Bi = \frac{h}{\lambda_f}$: Biot number, $\alpha = \frac{T_w}{T_e}$: Ratio of wall temperature to temperature at edge

3.2 Solution by Iteration Perturbation Method

In order to solve equations (28) – (31) using iteration perturbation method (where details can be found in [4]), we consider equations (29) and (30) when $\upsilon_f + \upsilon_0 = 1$.

Ayeni [1] has shown that $\exp\left(\frac{\theta}{1+\in\theta}\right)$ can be approximated as $1+(e-2)\theta+\theta^2$. In this paper we are going to take an approximation of the form

$$\exp\left(\frac{\theta}{1+\in\theta}\right)\approx 1+(e-2)\theta \tag{32}$$

Now we begin with the initial approximate solution:

$$f_0(\eta)=\eta-\frac{1}{b}\left(1-e^{-b\eta}\right)-\frac{1}{4}e^{-b\eta}+\frac{1}{8}e^{-2b\eta}+\frac{1}{8}, \tag{33}$$

where b is an unknown constant.

Equations (28) – (30) can be approximated by the following equations:

(34)
$$f'''+\left(\eta-\frac{1}{b}\left(1-e^{-b\eta}\right)-\frac{1}{4}e^{-b\eta}+\frac{1}{8}e^{-2b\eta}+\frac{1}{8}\right)f''+1-f'^2+\frac{\in}{(1+\in\theta)}f''\theta'=0$$

(35)
$$\varphi''+Sc\left(\eta-\frac{1}{b}\left(1-e^{-b\eta}\right)-\frac{1}{4}e^{-b\eta}+\frac{1}{8}e^{-2b\eta}+\frac{1}{8}\right)\varphi'+\frac{\in}{(1+\in\theta)}\varphi'\theta'+Sc\sigma\varphi\left(1+(e-2)\theta\right)=0$$

(36)
$$\theta''+\Pr\left(\eta-\tfrac{1}{b}\left(1-e^{-b\eta}\right)-\tfrac{1}{4}e^{-b\eta}+\tfrac{1}{8}e^{-2b\eta}+\tfrac{1}{8}\right)\theta'+\tfrac{\in}{(1+\in\theta)}\theta'^2-$$
$$\tfrac{\Pr Ec}{\in}\left(f'^3+\left(\eta-\tfrac{1}{b}\left(1-e^{-b\eta}\right)-\tfrac{1}{4}e^{-b\eta}+\tfrac{1}{8}e^{-2b\eta}+\tfrac{1}{8}\right)f'f''-f'f'''-f''^2\right)+$$
$$\tfrac{\Pr Ec}{(1+\in\theta)}f'f''\theta'-\Pr\delta\varphi\left(1+(e-2)\theta\right)=0$$

We rewrite equations (34) - (36) in the form:

(37)
$$f'''+bf''+\left(\eta-\frac{1}{b}\left(1-e^{-b\eta}\right)-\frac{1}{4}e^{-b\eta}+\frac{1}{8}e^{-2b\eta}+\frac{1}{8}-b\right)f''+1-f'^2+\frac{\in}{(1+\in\theta)}f''\theta'=0$$

(38)
$$\varphi''+Sc\varphi'+Sc\left(\eta-\tfrac{1}{b}\left(1-e^{-b\eta}\right)-\tfrac{1}{4}e^{-b\eta}+\tfrac{1}{8}e^{-2b\eta}+\tfrac{1}{8}-1\right)\varphi'+\tfrac{\in}{(1+\in\theta)}\varphi'\theta'+$$
$$Sc\sigma\varphi\left(1+(e-2)\theta\right)=0$$

(39)
$$\theta''+\Pr\theta'+\Pr\left(\eta-\tfrac{1}{b}\left(1-e^{-b\eta}\right)-\tfrac{1}{4}e^{-b\eta}+\tfrac{1}{8}e^{-2b\eta}+\tfrac{1}{8}-1\right)\theta'+\tfrac{\in}{(1+\in\theta)}\theta'^2-$$
$$\tfrac{\Pr Ec}{\in}\left(f'^3+\left(\eta-\tfrac{1}{b}\left(1-e^{-b\eta}\right)-\tfrac{1}{4}e^{-b\eta}+\tfrac{1}{8}e^{-2b\eta}+\tfrac{1}{8}\right)f'f''-f'f'''-f''^2\right)+$$
$$\tfrac{\Pr Ec}{(1+\in\theta)}f'f''\theta'-\Pr\delta\varphi\left(1+(e-2)\theta\right)=0$$

We let $1=\in^2 a$, $\sigma=\in p$, $\delta=\in c$ and embed an artificial parameter $\in$ in equations (37) – (39) as follows:

(40)
$$f'''+bf''+\in\left(\eta-\frac{1}{b}\left(1-e^{-b\eta}\right)-\frac{1}{4}e^{-b\eta}+\frac{1}{8}e^{-2b\eta}+\frac{1}{8}-b\right)f''+\in^2 a-f'^2+\frac{\in}{(1+\in\theta)}f''\theta'=0$$

(41)
$$\varphi'' + Sc\varphi' + \in Sc\left(\eta - \tfrac{1}{b}\left(1 - e^{-b\eta}\right) - \tfrac{1}{4}e^{-b\eta} + \tfrac{1}{8}e^{-2b\eta} + \tfrac{1}{8} - 1\right)\varphi' + \tfrac{\in}{(1+\in\theta)}\varphi'\theta' + \in Scp\varphi\left(1 + (e-2)\,\theta\right) = 0$$

(42)
$$\theta'' + \Pr\theta' + \in \Pr\left(\eta - \tfrac{1}{b}\left(1 - e^{-b\eta}\right) - \tfrac{1}{4}e^{-b\eta} + \tfrac{1}{8}e^{-2b\eta} + \tfrac{1}{8} - 1\right)\theta' + \tfrac{\in}{(1+\in\theta)}\theta'^2 - \tfrac{\Pr Ec}{\in}\left(f'^3 + \left(\eta - \tfrac{1}{b}\left(1 - e^{-b\eta}\right) - \tfrac{1}{4}e^{-b\eta} + \tfrac{1}{8}e^{-2b\eta} + \tfrac{1}{8}\right)f'f'' - f'f''' - f''^2\right) + \tfrac{\Pr Ec}{(1+\in\theta)}f'f''\theta' - \in \Pr c\varphi\left(1 + (e-2)\,\theta\right) = 0$$

Suppose that the solution of equations (40) – (42) can be expressed as:

$$\left.\begin{array}{c} f(\eta) = \in f_0(\eta) + \in^2 f_1(\eta) + ... \\ \varphi(\eta) = \varphi_0(\eta) + \in \varphi_1(\eta) + ... \\ \theta(\eta) = \theta_0(\eta) + \in \theta_1(\eta) + ... \end{array}\right\} \tag{43}$$

Substituting (43) into (40) – (42) and processing, we obtain

$$f_0''' + bf_0'' = 0, \quad f_0(0) = 0, \quad f_0'(0) = 0, \quad f_0'(\eta \to \infty) = \frac{1}{\in} \tag{44}$$

$$\varphi_0'' + Sc\varphi_0' = 0, \quad \varphi_0'(0) = 0, \quad \varphi_0(\eta \to \infty) = 1 \tag{45}$$

$$\theta_0'' + \Pr\theta_0' = 0, \quad \theta_0'(0) = \frac{Bi}{\in}(1-\alpha), \quad \theta_0(\eta \to \infty) = 0 \tag{46}$$

(47)
$$f_1''' + bf_1'' + \theta_0 f_0''' + b\theta_0 f_0'' + \left(\eta - \tfrac{1}{b}\left(1 - e^{-b\eta}\right) - \tfrac{1}{4}e^{-b\eta} + \tfrac{1}{8}e^{-2b\eta} + \tfrac{1}{8} - b\right)f_0'' + a - f_0'^2 + f_0''\theta_0' = 0, \quad f_0(0) = 0, \quad f_0'(0) = 0, \quad f_0'(\eta \to \infty) = 0$$

(48)
$$\varphi_1'' + Sc\varphi_1' + \theta_0\varphi_0'' + Sc\theta_0\varphi_0' + Sc\left(\eta - \tfrac{1}{b}\left(1 - e^{-b\eta}\right) - \tfrac{1}{4}e^{-b\eta} + \tfrac{1}{8}e^{-2b\eta} + \tfrac{1}{8} - 1\right)\varphi_0' + \varphi_0'\theta_0' + Scp\varphi_0\left(1 + (e-2)\,\theta_0\right) = 0, \quad \varphi_0'(0) = 0, \quad \varphi_0(\eta \to \infty) = 0$$

(49)
$$\theta_1'' + \Pr\theta_1' + \theta_0\theta_0'' + \Pr\theta_0\theta_0' + \Pr\left(\eta - \tfrac{1}{b}\left(1 - e^{-b\eta}\right) - \tfrac{1}{4}e^{-b\eta} + \tfrac{1}{8}e^{-2b\eta} + \tfrac{1}{8} - 1\right)\theta_0' + \theta_0'^2 - \Pr Ec\left(\left(\eta - \tfrac{1}{b}\left(1 - e^{-b\eta}\right) - \tfrac{1}{4}e^{-b\eta} + \tfrac{1}{8}e^{-2b\eta} + \tfrac{1}{8}\right)f_0'f_0'' - f_0'f_0''' - f_0''^2\right) - \Pr c\varphi_0\left(1 + (e-2)\,\theta_0\right) = 0, \quad \theta_1'(0) = 0, \quad \theta_1(\eta \to \infty) = 0$$

(50)
$$\varphi_2'' + Sc\varphi_2' + \theta_0\varphi_1'' + Sc\theta_0\varphi_1' + \theta_1\varphi_0'' + Sc\theta_1\varphi_0' + \varphi_1'\theta_0' + \varphi_0'\theta_1' + Sc\left(\eta - \tfrac{1}{b}\left(1 - e^{-b\eta}\right) - \tfrac{1}{4}e^{-b\eta} + \tfrac{1}{8}e^{-2b\eta} + \tfrac{1}{8} - 1\right)\varphi_1' + Sc\left(\eta - \tfrac{1}{b}\left(1 - e^{-b\eta}\right) - \tfrac{1}{4}e^{-b\eta} + \tfrac{1}{8}e^{-2b\eta} + \tfrac{1}{8} - 1\right)\theta_0\varphi_0' + Scp\varphi_0\theta_0\left(1 + (e-2)\,\theta_0\right) + Scp\varphi_0\theta_1\left(1 + (e-2)\,\theta_0\right) + Scp\varphi_1\left(1 + (e-2)\,\theta_0\right) + Scp\varphi_0(e-2)\,\theta_1 = 0, \quad \varphi_2'(0) = 0, \quad \varphi_2(\eta \to \infty) = 0$$

Seeking direct integration, we obtain the solution of equations (44) - (50) as

(51)

$$f(\eta) = \left(\eta - \frac{1}{b}\left(1 - e^{-b\eta}\right)\right) + \frac{\in^2}{2b}\left(\frac{1}{\in^2} - a\right)\eta^2 + \frac{n\in^2}{b^2}e^{-b\eta} + \frac{bm_1\in}{\Pr(b+\Pr)^2}e^{-(b+\Pr)\eta} - \frac{\in^2}{2b^3}\left(\frac{b}{4\in} + \frac{1}{\in^2} - \frac{1}{\in}\right)e^{-2b\eta} + \frac{\in}{144b^2}e^{-3b\eta} + \frac{\in^2}{b^2}\left(\frac{7b}{8\in} - \frac{2}{\in^2} + \frac{1}{\in}\right)\eta e^{-b\eta} + \frac{2\in^2}{b^3}\left(\frac{7b}{8\in} - \frac{2}{\in^2} + \frac{1}{\in}\right)e^{-b\eta} - \frac{\in}{2b}\eta^2 e^{-b\eta} - \frac{2\in}{b^2}\eta e^{-b\eta} - \frac{3\in^2}{b^3}e^{-b\eta} + \in^2\left(-\frac{n}{b^2} - \frac{bm_1}{\in\Pr(b+\Pr)^2} + \frac{1}{2b^3}\left(\frac{b}{4\in} + \frac{1}{\in^2} - \frac{1}{\in}\right) - \frac{1}{144\in b^2} - \frac{2}{b^3}\left(\frac{7b}{8\in} - \frac{2}{\in^2} + \frac{1}{\in}\right) + \frac{3}{b^3}\right)$$

(52)

$$\theta(\eta) = me^{-\Pr\eta} + \in\left(\begin{array}{l} -p_0 e^{-\Pr\eta} - p_1 e^{-2\Pr\eta} - p_2\left(\eta e^{-\Pr\eta} + \frac{1}{\Pr}e^{-\Pr\eta}\right) - p_3\left(\begin{array}{l}\eta^2 e^{-\Pr\eta} + \frac{2}{\Pr}\eta e^{-\Pr\eta} + \\ \frac{2}{\Pr^2}e^{-\Pr\eta}\end{array}\right) - \\ p_4\eta e^{-(b+\Pr)\eta} + (p_5 + p_6)e^{-(b+\Pr)\eta} + p_7 e^{-(2b+\Pr)\eta} - p_8 e^{-(3b+\Pr)\eta}\end{array}\right)$$

(53)

$$\varphi(\eta) = 1 + \in^2\left(\begin{array}{l} -\frac{t_0}{\Pr}e^{-\Pr\eta} + \frac{t_0}{Sc}e^{-Sc\eta} - \frac{t_1}{2\Pr}e^{-2\Pr\eta} + \frac{t_1}{Sc}e^{-Sc\eta} - \frac{t_2}{\Pr}\eta e^{-\Pr\eta} - \frac{t_2}{\Pr^2}e^{-\Pr\eta} - \\ \frac{t_3}{\Pr}e^{-\Pr\eta} + \frac{t_3}{Sc}e^{-Sc\eta} - \frac{t_4}{\Pr}\left(\eta^2 e^{-\Pr\eta} + \frac{2}{\Pr}\eta e^{-\Pr\eta} + \frac{2}{\Pr^2}e^{-\Pr\eta}\right) - \frac{t_5}{\Pr}\eta e^{-\Pr\eta} - \\ -\frac{t_5}{\Pr^2}e^{-\Pr\eta} - \frac{t_6}{\Pr}e^{-\Pr\eta} + \frac{t_6}{Sc}e^{-Sc\eta} - \frac{t_7}{(b+\Pr)}\eta e^{-(b+\Pr)\eta} - \frac{t_7}{(b+\Pr)^2}e^{-(b+\Pr)\eta} - \\ \frac{t_8}{(b+\Pr)}e^{-(b+\Pr)\eta} + \frac{t_8}{Sc}e^{-Sc\eta} - \frac{t_9}{(b+\Pr)}e^{-(b+\Pr)\eta} + \frac{t_9}{Sc}e^{-Sc\eta} - \frac{t_{10}}{(2b+\Pr)}e^{-(2b+\Pr)\eta} + \\ \frac{t_{10}}{Sc}e^{-Sc\eta} - \frac{t_{11}}{(3b+\Pr)}e^{-(3b+\Pr)\eta} + \frac{t_{11}}{Sc}e^{-Sc\eta} - \frac{t_{12}}{3\Pr}e^{-3\Pr\eta} + \frac{t_{12}}{Sc}e^{-Sc\eta} - \frac{t_{13}}{2\Pr}\eta e^{-2\Pr\eta} - \\ \frac{t_{13}}{4\Pr^2}e^{-2\Pr\eta} - \frac{t_{14}}{2\Pr}e^{-2\Pr\eta} + \frac{t_{14}}{Sc}e^{-Sc\eta} - \frac{t_{15}}{2\Pr}\left(\eta^2 e^{-2\Pr\eta} + \frac{1}{\Pr}\eta e^{-\Pr\eta} + \frac{1}{2\Pr^2}e^{-\Pr\eta}\right) \\ -\frac{t_{16}}{2\Pr}\eta e^{-2\Pr\eta} - \frac{t_{16}}{4\Pr^2}e^{-2\Pr\eta} - \frac{t_{17}}{2\Pr}e^{-2\Pr\eta} + \frac{t_{17}}{Sc}e^{-Sc\eta} - \frac{t_{18}}{(b+2\Pr)}\eta e^{-(b+2\Pr)\eta} - \\ \frac{t_{18}}{(b+2\Pr)^2}e^{-(b+2\Pr)\eta} - \frac{t_{19}}{(b+2\Pr)}e^{-(b+2\Pr)\eta} + \frac{t_{19}}{Sc}e^{-Sc\eta} - \frac{t_{20}}{(b+2\Pr)}e^{-(b+2\Pr)\eta} + \frac{t_{20}}{Sc}e^{-Sc\eta} \\ -\frac{t_{21}}{(2b+2\Pr)}e^{-(2b+2\Pr)\eta} + \frac{t_{21}}{Sc}e^{-Sc\eta} - \frac{t_{22}}{(3b+2\Pr)}e^{-(3b+2\Pr)\eta} + \frac{t_{19}}{Sc}e^{-Sc\eta} - \\ \left(\begin{array}{l} -\frac{t_0}{\Pr} + \frac{t_0}{Sc} - \frac{t_1}{2\Pr} + \frac{t_1}{Sc} - \frac{t_2}{\Pr^2} - \frac{t_3}{\Pr} + \frac{t_3}{Sc} - \frac{t_4}{\Pr}\left(\frac{2}{\Pr^2}\right) - \frac{t_5}{\Pr^2} - \frac{t_6}{\Pr} + \frac{t_6}{Sc} - \frac{t_7}{(b+\Pr)^2} - \\ \frac{t_8}{(b+\Pr)} + \frac{t_8}{Sc} - \frac{t_9}{(b+\Pr)} + \frac{t_9}{Sc} - \frac{t_{10}}{(2b+\Pr)} + \frac{t_{10}}{Sc} - \frac{t_{11}}{(3b+\Pr)} + \frac{t_{11}}{Sc} - \frac{t_{12}}{3\Pr} + \frac{t_{12}}{Sc} - \\ \frac{t_{13}}{4\Pr^2} - \frac{t_{14}}{2\Pr} + \frac{t_{14}}{Sc} - \frac{t_{15}}{2\Pr}\left(\frac{1}{2\Pr^2}\right) - \frac{t_{16}}{4\Pr^2} - \frac{t_{17}}{2\Pr} + \frac{t_{17}}{Sc} - \frac{t_{18}}{(b+2\Pr)^2} - \frac{t_{19}}{(b+2\Pr)} \\ +\frac{t_{19}}{Sc} - \frac{t_{20}}{(b+2\Pr)} + \frac{t_{20}}{Sc} - \frac{t_{21}}{(2b+2\Pr)} + \frac{t_{21}}{Sc} - \frac{t_{22}}{(3b+2\Pr)} + \frac{t_{22}}{Sc}\end{array}\right)\end{array}\right),$$

where

$$m = \frac{Bi}{\in \Pr}\left(1-\alpha\right), \quad m_1 = \frac{Bi}{\in}\left(1-\alpha\right), \quad a = \frac{1}{\in^2}, \quad p = \frac{\sigma}{\in}, \quad q_0 = Scpm,$$

$$n = b\left(\frac{1}{\in b^2} - \frac{1}{48\in b} - \frac{bm_1}{\in \Pr(b+\Pr)} - \frac{1}{b^2}\left(\frac{7b}{8\in} - \frac{2}{\in^2} + \frac{1}{\in}\right) + \frac{1}{b^2}\left(\frac{b}{4\in} + \frac{1}{\in^2} - \frac{1}{\in}\right)\right),$$

$$p_0 = \frac{1}{\Pr}\left(\begin{array}{l} \frac{\Pr m_1}{b}\left(\frac{1}{b} - \frac{1}{4}\right) - \frac{m_1^2}{\Pr} + \frac{\Pr m_1}{16b} - \frac{\Pr Ec}{24\in^2} + \frac{\Pr Ec}{b\in^2} + \frac{\Pr Ec}{2b\in^2}\left(\frac{3b}{8} - 1\right) + \\ \frac{\Pr Ec}{b\in^2}\left(2 - 2b^2 - \frac{3b}{8}\right) \end{array}\right),$$

$$p_1 = \frac{m_1^2}{2\Pr^2}, \quad p_2 = \left(\frac{Ec}{\in^2}\left(b^2 + \frac{b}{8} - 1\right) - m_1\left(\frac{1}{b} + \frac{7}{8}\right)\right), \quad p_3 = \left(\frac{Ecb}{2\in^2} - \frac{m_1}{2}\right),$$

$$p_4 = \frac{\Pr Ec}{(b+\Pr)\in^2}, \quad p_5 = \frac{\Pr Ec}{(b+\Pr)^2\in^2}, \quad q_1 = Scpm^2\left(e-2\right),$$

$$p_6 = \left(\frac{\Pr m_1}{b(b+\Pr)}\left(\frac{1}{b} - \frac{1}{4}\right) + \frac{\Pr Ec}{b(b+\Pr)\in^2} + \frac{\Pr Ec}{b(b+\Pr)\in^2}\left(2 - 2b^2 - \frac{3b}{8}\right)\right),$$

$$p_7 = \left(\frac{\Pr m_1}{16b(2b+\Pr)} + \frac{\Pr Ec}{2b(b+\Pr)\in^2}\left(\frac{3b}{8} - 1\right)\right), \quad p_8 = \frac{\Pr Ec}{24(3b+\Pr)\in^2},$$

$$q_2 = -Scpp_0\left(e-1\right), \quad q_3 = -Scpp_1\left(e-1\right), \quad q_4 = -Scpp_2\left(e-1\right),$$

$$q_5 = -Scpp_3\left(e-1\right), \quad q_6 = Scpp_4\left(e-1\right), \quad q_7 = Scpp_5\left(e-1\right),$$

$$q_8 = Scpp_6\left(e-1\right), \quad q_9 = Scpp_7\left(e-1\right), \quad q_{10} = -Scpp_8\left(e-1\right),$$

$$q_{11} = -Scpmp_0\left(e-2\right), \quad q_{12} = -Scpmp_1\left(e-2\right), \quad q_{13} = -Scpmp_2\left(e-2\right),$$

$$q_{14} = -Scpmp_3\left(e-2\right), \quad q_{15} = Scpmp_4\left(e-2\right), \quad q_{16} = Scpmp_5\left(e-2\right),$$

$$q_{17} = Scpmp_6\left(e-2\right), \quad q_{18} = Scpmp_7\left(e-2\right), \quad q_{19} = -Scpmp_8\left(e-2\right),$$

$$m_2 = \left(q_0 + q_3 + \frac{q_4}{\Pr} + \frac{2q_5}{\Pr^2}\right), \quad m_3 = \left(q_1 + q_3 + q_{11} + \frac{q_{13}}{\Pr} + \frac{2q_{14}}{\Pr^2}\right), \quad m_5 = \left(q_7 + q_8\right),$$

$$m_6 = \left(q_{13} + \frac{2q_{14}}{\Pr}\right), \quad m_7 = \left(q_{16} + q_{17}\right), \quad t_0 = \frac{m_2}{(\Pr - Sc)}, \quad t_1 = \frac{m_3}{(2\Pr - Sc)},$$

$$t_2 = \frac{m_4}{(\Pr - Sc)}, \quad t_3 = \frac{m_4}{(\Pr - Sc)^2}, \quad t_4 = \frac{q_5}{(\Pr - Sc)},$$

$$t_5 = \frac{2q_5}{(\Pr - Sc)^2}, \quad t_6 = \frac{2q_5}{(\Pr - Sc)^3}, \quad t_7 = \frac{q_6}{(b+\Pr - Sc)},$$

$$t_8 = \frac{q_6}{(b+\Pr - Sc)^2}, \quad t_9 = \frac{m_5}{(b+\Pr - Sc)}, \quad t_{10} = \frac{q_9}{(2b+\Pr - Sc)},$$

$$t_{11} = \frac{q_{10}}{(3b+\Pr - Sc)}, \quad t_{12} = \frac{q_{12}}{(3\Pr - Sc)}, \quad t_{13} = \frac{m_6}{(2\Pr - Sc)},$$

$$t_{14} = \frac{m_6}{(2\Pr - Sc)^2}, \quad t_{15} = \frac{q_{14}}{(2\Pr - Sc)}, \quad t_{16} = \frac{2q_{14}}{(2\Pr - Sc)^2},$$

$$t_{17} = \frac{2q_{14}}{(2\Pr - Sc)^3}, \quad t_{18} = \frac{q_{15}}{(b+2\Pr - Sc)}, \quad t_{19} = \frac{q_{15}}{(b+2\Pr - Sc)^2},$$

$$t_{20} = \frac{m_7}{(b+2\Pr - Sc)}, \quad t_{21} = \frac{q_{18}}{(2b+2\Pr - Sc)}, \quad t_{22} = \frac{q_{19}}{(3b+2\Pr - Sc)},$$

The computations were done using computer symbolic algebraic package MAPLE.

4. Results and Discussion

The systems of partial differential equations describing a laminar premixed flame impinges on a plane solid surface in the presence of work due to compression and heat generated due to viscous dissipation are solved analytically using a similarity transformation and iteration perturbation method. The numerical values of Skin Friction, Nusselt Number and Sherwood Number are arranged in Table 1 below for various values of the parameters involved. Analytical solutions of equations (28) - (31) are computed for the values of $\Pr = 0.71, 0.85, 1.00,\ \ Sc = 0.22, 0.62, 0.78,\ \ Ec = 0.2, 0.6, 1.0,\ \ Bi = 10, 20, 30,\ \ \alpha = 1.2,\ \ \sigma = 0.001, \in = 0.01,\ \ b = 0.3062$. The following figures explain the fluid velocity, flame temperature and species concentration distribution against different dimensionless parameters.

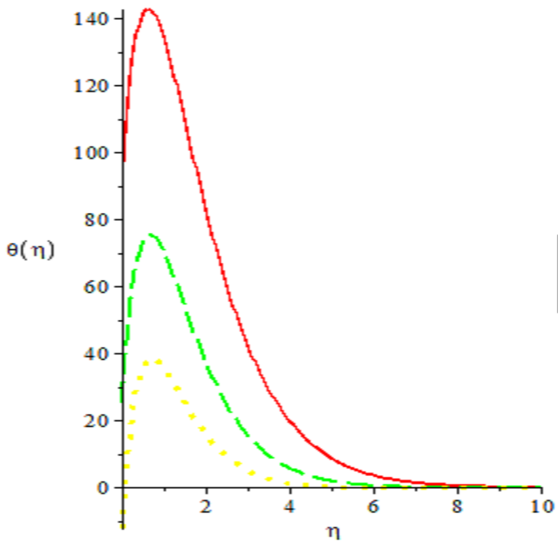

Figure 1: Variation of temperature with Pr

From figure 1, we can conclude that with the increase of Prandtl number (Pr), maximum temperature decreases.

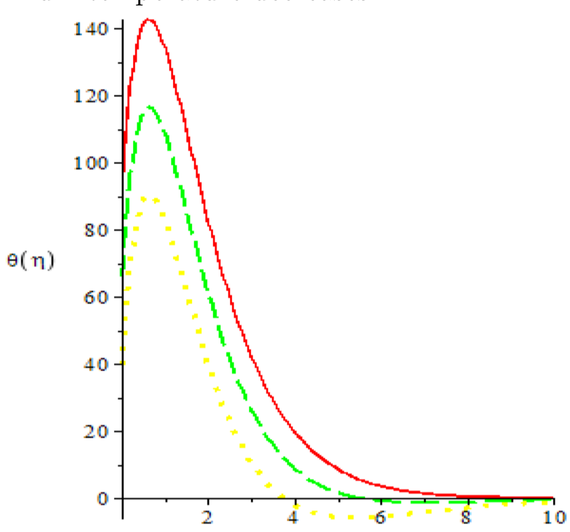

Figure 2: Variation of temperature with Ec

From figure 2, we can conclude that with the increase of Eckert number (Ec), temperature decreases.

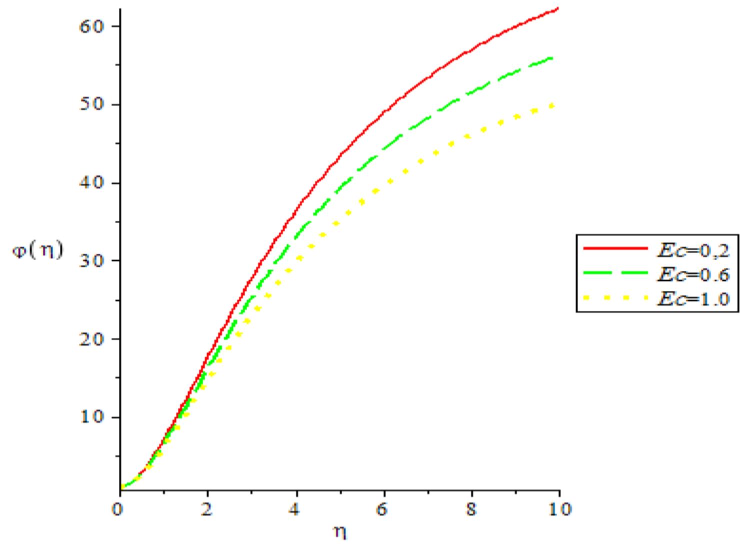

Figure 3: Variation of species concentration with Ec

From figure 3, we can conclude that with the increase of Eckert number (Ec), species concentration decreases.

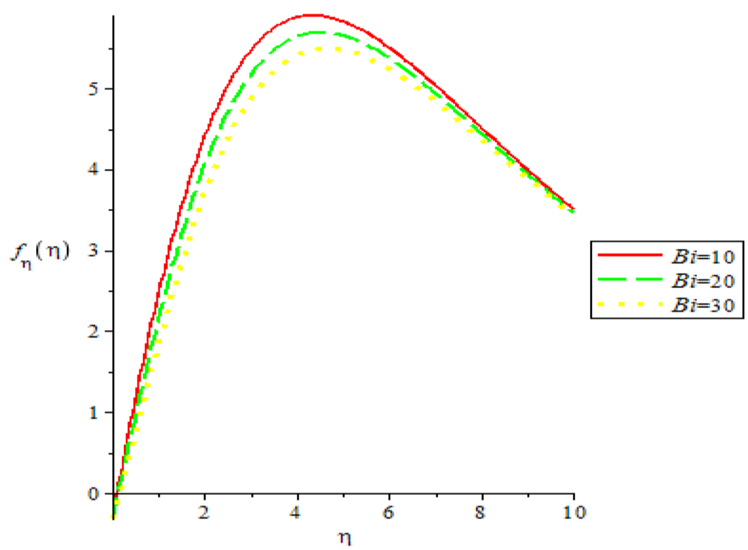

Figure 4: Variation of velocity with Bi

From figure 4, we can conclude that with the increase of Biot number (Bi), velocity decreases.

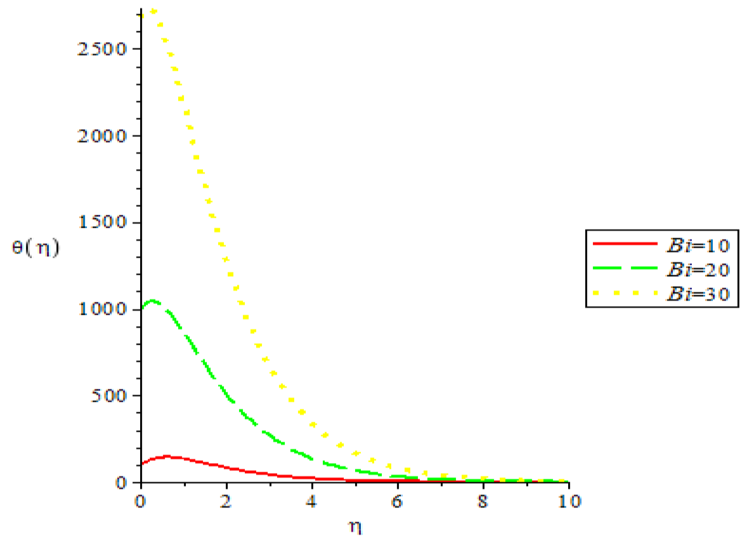

Figure 5: Variation of temperature with Bi

From figure 5, we can conclude that with the increase of Biot number (Bi), temperature increases.

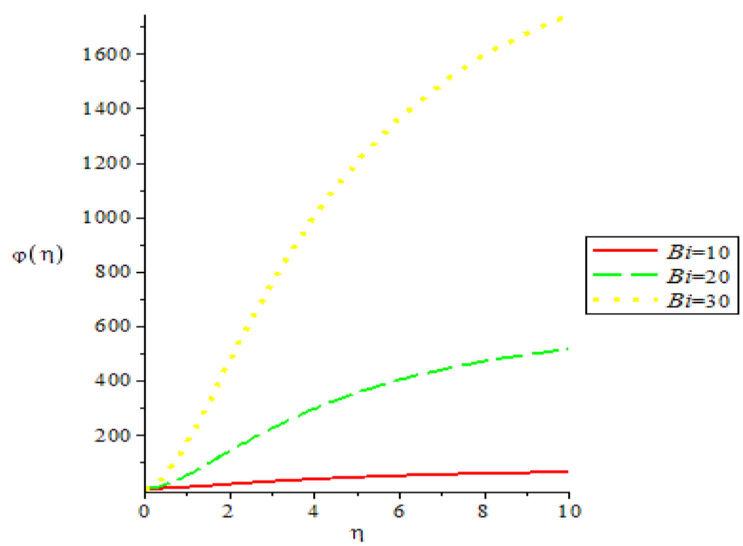

Figure 6: Variation of species concentration with Bi

From figure 6, we can conclude that with the increase of Biot number (Bi), species concentration increases.

Table 1: The numerical values of skin friction, rate of heat and mass transfer.

Bi	Ec	Pr	Sc	$f''(0)$	$\theta'(0)$	$\varphi'(0)$
10	0.2	0.71	0.22	3.088222533	200.0000001	0.000000008
10	0.2	0.85	0.22	3.161193663	200.0000001	0.000000002
10	0.2	1.00	0.22	3.222018895	200.0000001	0.000000002
10	0.2	0.71	0.62	3.088222533	200.0000001	0.000000020
10	0.2	0.71	0.78	3.088222533	200.0000001	-0.00000004
10	0.6	0.71	0.22	3.088222533	65.9991809	-0.00000001
10	1.0	0.71	0.22	3.088222533	39.6324302	0.000000000
20	0.2	0.71	0.22	2.485585251	400.0000008	0.000000053
30	0.2	0.71	0.22	1.882947980	599.9999992	0.000000063

5. Conclusion

From the studies made on this paper we conclude as under.

(1) Prandtl number decreases the flame temperature.
(2) Eckert number decreases the flame temperature and species concentration.
(3) Biot number decreases the fluid velocity and enhances the flame temperature and species concentration.
(4) Shear stress increases due to increase of Prandtl number.
(5) Heat transfer rate decreases due to increase of Eckert number.
(6) Shear stress decreases for increase of Biot number.
(7) Heat transfer rate as well as mass transfer rate increases due to increase of Biot number.

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