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Radiative Effects on Heat and Mass Transfer of Magnetohydrodynamics Fluid Flow in Porous Media

S. A. Amoo

Abstract

Magnetohydrodynamics (MHD) flow involving heat and mass transfer under the influence of radiation, heat source, and chemical reaction effects is of great concern in physical sciences and engineering. The gap in earlier studies showed that few parameters emanating from studies like this, due to difficulty usually encountered in reaching quick convergences when solving such problems. The study therefore, investigated radiative heat and mass transfer of MHD fluid flow effects in porous media. The Partial Differential Equations (PDEs) formulated were reduced to a set of nonlinear Ordinary Differential Equations (ODEs) by similarity transformations which were solved using fourth-order Runge-Kutta method with shooting technique using Maple 18 software. The study concluded that the increase in radiation and heat generation had significant effects on the MHD fluid flow in porous media over stretching surface. The study therefore recommended the use of increased renewable energy in metallurgical applications, plastic extrusion and MHD power generation systems.

1. Introduction

Research investigation in radiative effects on heat and mass transfer of MHD fluid flow in porous media remains within the core of engineering, applied technology, physics and mathematics. This is so because of its dynamics. All advances in media for delivery of such studies provide challenges to the academics. These challenges among others included but not limited to day-in-day out research and formulation of problems required solutions and applications.

Many problems emanate from various fields that might be in form of partial differential equations in which task of solving them play important roles due to their applications in

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Department of Mathematics and Statistics, Federal University Wukari, Wukari, Nigeria
E-mails of the corresponding author: amoo@fuwukari.edu.ng
ORCID of the corresponding author: 0000-0002-0874-0842

real life. The dynamic nature of such applications in real life too poses problems if not well applied. The dynamic nature of a study like this has applications in environmental sciences, health sciences, geophysics, oceanography, meteorology, atmosphere, health sciences, civil engineering, mechanical, chemical, computer engineering as well as industries related in heat and mass transfer phenomena. It must be noted that MHD fluid flow in porous media has wide and numerous applications in industry and environments. Some of other areas of applications but not limited are : flow of ground water through soil and rocks (porous media) that are very important for agriculture and pollution control; extraction of oil and natural gas from rocks which are prominent in oil and gas industries; functioning of tissues in body (bone, cartilage and muscle and so on) belong to porous media, flow of blood and treatments through them; understanding various medical conditions (such as tumor growth, a formation of porous media) and their treatment (such as injection, a flow through porous media in medical sciences). In the similar manner, the subject of MHD has attracted the attention of a large number of scholars due to its diverse applications in several problems of technological importance

In literature, [5] obtained numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation. [15] dwelled on MHD heat and mass transfer over moving vertical plate with a convective surface boundary condition. [19] presented the Soret and Dufour Effects on Mixed Convection from an Exponentially Stretching Surface, with in the same period, [12] numerically analysed MHD Boundary layer flow due to an exponentially stretching sheet with radiation effects. The work of [20] on Soret and Dufour effect on MHD free convective heat and mass transfer flow over a stretching vertical plate with suction and heat source/sink presented. [16] obtained the Mathematics Structure and Analysis of an MHD Flow in Porous Media.

[14] evaluated MHD boundary layer flow of heat and mass transfer over a moving vertical plate in a porous medium with suction and viscous dissipation. [18] obtained analysis of boundary layer phenomena of MHD flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium. [13] obtained steady MHD free convective flow and heat transfer over nonlinearly stretching sheet embedded in an extended Darcy-Forcheimmer Porous Medium with Viscous Dissipation. [6] analysed radiation and mass transfer effects on MHD boundary layer sheet with heat source. On this note too, [11] obtained numerical effects of thermal radiation on heat and mass transfer over an exponentially-porous stretching surface. [9] presented the effect of heat generation and thermal radiation on MHD flow near a stagnation point on a linear stretching sheet in porous medium and presence of variable thermal conductivity and mass transfer. [4] numerically analysed MHD flow of heat and mass transfer embedded in thermally stratified media over exponentially stretching Sheet. [3] obtained nonlinear MHD boundary layer flow embedded in Darcy-Forcheimmer porous medium with viscous dissipation and chemical reaction. [1] evaluated thermal radiation effect on heat and mass transfer of MHD flow in porous media over an exponentially-stretching surface. In considering research applications, [2] obtained MHD heat and mass transfer in porous media to analyse issues of environmental remediation and restoration.

In this paper, variables like thermal radiation, heat source, heat flux, thermal diffusion (dufour), diffusion thermal (soret), chemical reaction, concentration of fluid, fluid viscosity were reported.

1.1. **Statement of the Problem.** MHD fluid flow is essential in engineering and industrial processes and can be translated into mathematical models. To forecast the behaviour and distinctiveness of heat and mass transfer of such fluids accurately, it becomes essential to study the velocity profiles, temperature parameters and concentration behaviour of the flow, hence the rate of heat absorption or generation of such a system with chemical reactions. Also, the study of fluid flow through Darcy permeable media with radiation, Dufour and Soret effects were incorporated under different angle of inclination of the surface. The study therefore, investigated radiative heat and mass transfer of MHD fluid flow in porous media over exponentially-stretching-surface.

1.2. **Justification for the Study.** Fluid flow through or in the porous media constitute a vital area in fluid mechanics because of its applications in many systems. These range from its worth in human thermo-regulation, irrigation, hydrology, petroleum engineering, filtration, absorption in chemical processes, and climate changes.Again, MHD flow by the influence of heat source, chemical reaction, Soret and Dufour effects is of great concern in physical sciences and engineering. Also, the cross-diffusion in fluid-saturated porous media finds application in a variety of engineering processes such as chemical catalytic reactors andmetallurgical applications such as hot rolling of wires, drawing of metals and plastic extrusion, heat exchanger devices as well as MHD power generation systems. In the similar perspective, earlier studies limited investigation to few parameters, due to difficulty usually encountered in reaching quick convergences when solving similar problems. The study is important because of the reasons listed and for its applications. Therefore, it is necessary to investigate the variable characteristics, parameters and interactions that explain radiative effects on heat and mass transfer of MHD fluid flow in porous media

1.3. **Aim and Objectives of the Study.** The study aimed to present the analysis of variable characteristics that explain radiative heat and mass transfer of MHD fluid flow in porous media over exponentially-stretching surface.
The objectives of the study were to:

(1) Assess the impacts of thermal radiation parameter on heat and mass transfer flow in porous media
(2) Establish the influence of some parameters on vertically-stretching surface.
(3) Determine the effects of cross diffusion parameters on heat and mass transfer in porous media on inclined surface.

1.4. **Scope of the Study.** It is within the steady vertical and inclined stretching surfaces

2. Formulation of the Problem and Mathematical Analysis

Radiative effects on heat and mass transfer of MHD fluid flow of steady, viscous and incompressible in porous media over vertical and inclined surfaces considered two models. Model 1 is devoid of inclined component in the momentum equation, Dufour and Soret effects in energy and concentration equations respectively.
On either of the models, a uniform transverse variable magnetic field B(x) is applied perpendicular to the direction of the flow with chemical reactions taking place in the fluid flow. The plates were maintained at temperature and species concentration (T_w, and C_w) and the free steam temperature and specie ($T\infty$ and $C\ \infty$) respectively. The geometrical models and equations governing the radiative effects of heat and mass transfer of MHD fluid flow in porous media over exponentially-stretching surface were figures (2.1) and (2.2).

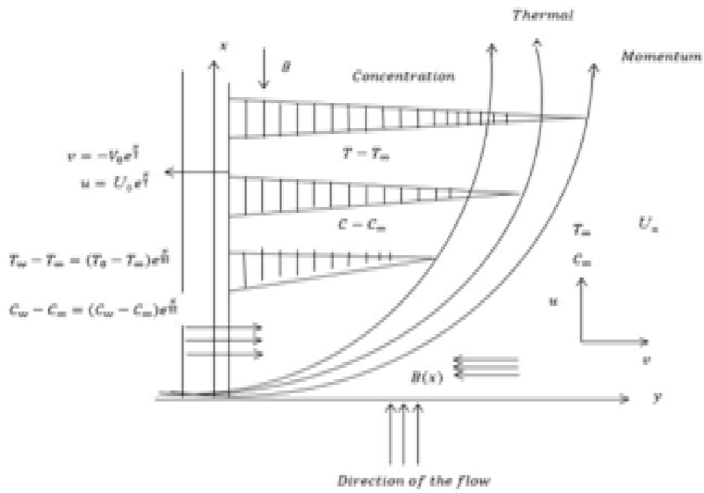

Figure 1: The Geometry of the Model and Coordinate System

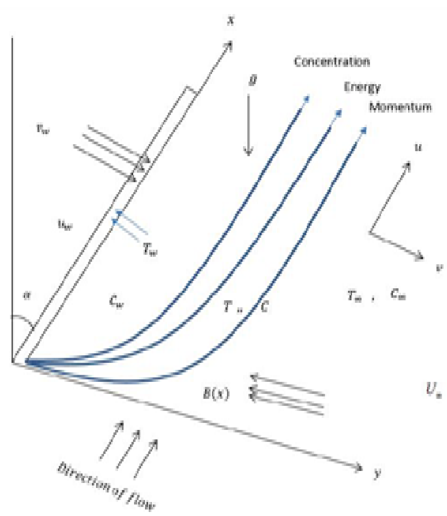

Figure 2: The Geometry of the Model and Coordinate System

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

Model 1:
Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{1}{\rho}\sigma {B_0}^2(x)u-\frac{\nu}{K}u+g\beta_T\left(T-T_\infty\right)+g\beta_C\left(C-C_\infty\right) \tag{2.2}$$

Energy equation

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+Q_0\left(T-T_\infty\right) \tag{2.3}$$

Concentration equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\gamma\left(C-C_\infty\right) \tag{2.4}$$

Model 2
Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+-\frac{1}{\rho}\sigma {B_0}^2(x)u-\frac{\nu}{K}u+g\beta_T\left(T-T_\infty\right)\cos(\alpha) \tag{2.5}$$
$$+g\beta_C\left(C-C_\infty\right)\cos(\alpha)$$

Energy equation

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+Q_0\left(T-T_\infty\right)+\frac{D_m\partial^2 C}{C_s\partial y^2} \tag{2.6}$$

Concentration equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\gamma\left(C-C_\infty\right)+\frac{D_m\partial^2 T}{T_m\partial y^2}, \tag{2.7}$$

subject to the following boundary conditions:

$$u=U_0e^{\frac{x}{L}},v=-V_0e^{\frac{x}{L}},T=T_w=T_\infty+T_0e^{\frac{x}{2L}},C=C_w=C_\infty+C_0e^{\frac{x}{2L}} \tag{2.8}$$

at

$$y=0,\ u\to 0,\ T\to T_\infty,\ C\to C_\infty\ as\ y\to\infty,$$

where u and v velocity component in the x direction, velocity component in the y direction C, and T are concentration of the fluid species and fluid temperature respectively. $L$is the reference length, $B(x)$ is the magnetic field strength, U_0 is the reference velocity and V_0 is the permeability of the porous surface. The physical quantities K; ρ, ν, σ,D, k, C_p, Q_0, γ, C_s , T_m and K_T are the permeability of the porous medium, density,fluid kinematics viscosity, electric conductivity of the fluid, coefficient of mass diffusivity,thermal conductivity of the fluid, specific heat capacity at constant pressure, rate of specific internal heat generation or absorption, reaction rate coefficient, concentration susceptibility and mean fluid temperature, and thermal diffusion ratio respectively. $g$is the gravitational acceleration, the last terms of equations (2.6) and (2.7) are Dufour or diffusion thermo effect and the Soret or thermo-diffusion effect. β_T and β_C are the thermal and mass expansion coefficients respectively. q_r is the radiative heat flux in the y direction. By using the Rossel and approximation according to [9], [1], the radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma_0}{3\delta}\frac{\partial T^4}{\partial y} \tag{2.9}$$

where σ_0 and δ are the Stefan-Boltzmann constant and the mean absorption coefficientrespectively. Assume the temperature difference within the flow is sufficiently small suchthat T^4 may be expressed as a linear function of temperature, using Taylor series toexpand T^4 about the free stream $T_\infty$and neglecting higher order terms, this gives theapproximation

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{2.10}$$

Substituting equations (2.8) and (2.9) in equation (2.3) and equation (2.6) respectively the equations (2.11) and (2.12) emerge:

$$\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \left(\frac{k}{\rho C_p} + \frac{16\sigma_0 T_\infty^3}{3\rho C_p \delta}\right)\frac{\partial^2 T}{\partial y^2} + Q_0\left(T - T_\infty\right) \tag{2.11}$$

$$\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \left(\frac{k}{\rho C_p} + \frac{16\sigma_0 T_\infty^3}{3\rho C_p \delta}\right)\frac{\partial^2 T}{\partial y^2} + Q_0\left(T - T_\infty\right) + \frac{D_m \partial^2 T}{C_s \partial y^2} \tag{2.12}$$

The magnetic field $B(x)$ is assumed to be in the form

$$B(x) = B_0 e^{\frac{x}{2L}} \tag{2.13}$$

where B_0 is the constant magnetic field. Introducing the stream function $\psi(x, y)$ such that:

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \tag{2.14}$$

Using equation (2.14) to substitute equation (2.1), Cauchy-Riemann equation was satisfied. Now substituting equation (2.14) in equations (2.2), (2.4) (2.5), (2.7), (2.11) and (2.12), the following equations emerge:

Model 1:

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi \partial^2 \psi}{\partial x \partial y^2} = \nu\frac{\partial^3 \psi}{\partial y^3} - \frac{\sigma}{\rho}B_0 e^{\frac{x}{2L}}\left(\frac{\partial \psi}{\partial y}\right) - \frac{\upsilon}{K}\left(\frac{\partial \psi}{\partial y}\right) + g\beta_T\left(T - T_\infty\right) + g\beta_C\left(C - C_\infty\right) \tag{2.15}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \left(\frac{k}{\rho C_p} + \frac{16\sigma_0 T_\infty^3}{3\rho C_p \delta}\right)\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p}\left(T - T_\infty\right) \tag{2.16}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - \gamma\left(C - C_\infty\right) \tag{2.17}$$

Model 2:

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi \partial^2 \psi}{\partial x \partial y^2} = \nu\frac{\partial^3 \psi}{\partial y^3} - \frac{\sigma}{\rho}B_0 e^{\frac{x}{2L}}\left(\frac{\partial \psi}{\partial y}\right) - \frac{\upsilon}{K}\left(\frac{\partial \psi}{\partial y}\right) + g\beta_T\left(T - T_\infty\right)\cos\left(\alpha\right) + g\beta_C\left(C - C_\infty\right)\cos\left(\alpha\right) \tag{2.18}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\left(\frac{k}{\rho C_p}+\frac{16\sigma_0 T_\infty^3}{3\rho C_p\delta}\right)\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}\left(T-T_\infty\right)+\frac{D_m\partial^2 T}{C_s\partial y^2} \tag{2.19}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\gamma\left(C-C_\infty\right)+\frac{D_m\partial^2 T}{T_m\partial y^2} \tag{2.20}$$

The corresponding boundary conditions become:

$$\frac{\partial\psi}{\partial y}=U_0 e^{\frac{x}{L}},\ \frac{\partial\psi}{\partial x}=V_0 e^{\frac{x}{L}},\ T=T_w=T_\infty+T_0 e^{\frac{x}{2L}},\ C=C_w=C_\infty+C_0 e^{\frac{x}{2L}} \tag{2.21}$$

at

$$y=0,\ \frac{\partial\psi}{\partial y}\to 0,\ T\to T_\infty,\ C\to C_\infty\ as\ \ y\ \to\ \ \infty$$

Applying similarity variable (2.22) on equations (2.15) to (2.20), according to ([1] & [17]):

$$\psi\left(x,\,y\right)\ =\ \sqrt{2\upsilon U_0 L e^{\frac{x}{2L}}}\,f\left(\eta\right),\ \ \eta\ =\ y\sqrt{\frac{U_0}{2\upsilon L}e^{\frac{x}{2L}}} \tag{2.22}$$

$$T=T_\infty+T_0 e^{\frac{x}{2L}}\theta\left(\eta\right),\ \ C=C_\infty+C_0 e^{\frac{x}{2L}}\phi\left(\eta\right)$$

The following defined transforms were obtained

$$\frac{\partial\psi}{\partial x}= \tag{2.23}$$

$$\frac{\partial T}{\partial x}= \tag{2.24}$$

$$\frac{\partial C}{\partial x}= \tag{2.25}$$

Substituting the defined transform (2.23) to (2.25) into equation (2.15) to (2.20) respectively, to obtain the system of ordinary differential equations for model 1 ((2.26) to (2.28)) and model 2 ((2.29) to (2.31)) respectively:

$$f^{'''}+ff^{''}-2f^{'2}-(M+D_a)f^{'}+G_r\theta+G_c\phi=0 \tag{2.26}$$

$$\left(1+\frac{4}{3}R\right)\theta^{''}+P_r(f\theta^{'}-f^{'}\theta+Q\theta)=0 \tag{2.27}$$

$$\phi^{''}+S_c(f\phi^{'}-f^{'}\phi-\lambda\phi)=0 \tag{2.28}$$

$$f^{'''}+ff^{''}-2f^{'2}-(M+D_a)f^{'}+G_r\theta\cos(\alpha)+G_c\phi\cos(\alpha)=0 \tag{2.29}$$

$$\left(1+\frac{4}{3}R\right)\theta^{''}+P_r(f\theta^{'}-f^{'}\theta+Q\theta+D_f\phi")=0 \tag{2.30}$$

$$\phi^{''}+S_c(f\phi^{'}-f^{'}\phi-\lambda\phi+S_r\theta^{''})=0 \tag{2.31}$$

where corresponding boundary conditions take the form equation (2.32):

$$f = f_w, f' = 1, \theta = 1, \phi = 1 \quad at \quad \eta = 0$$
$$f' = 0, \theta = 0, \phi = 0 \quad as \quad \eta \to \infty \tag{2.32}$$

where, α is the angle of inclination, $M =$ ....

The governing equations of radiative heat and mass transfer of MHD fluid flow in porous media were essentially nonlinear ordinary differential equations. Hence, the system of non-linear ODEs together with the boundary conditions was solved numerically using fourth order Runge-Kutta scheme with shooting technique. The method has been proven to be adequate for boundary layer equations, seems to give accurate results and has been widely used ([7] & [8]). In the numerical method employed in solving the models or equations i.e. the boundary valued problems (BPV).Shooting method reformulates the BVP to IVP by adding sufficient number of conditions at one end and adjust these conditions until the given conditions were satisfied at the other end while Runge-Kutta method solves the initial value problems. These methods are based on finite difference numerical techniques. Consider the two-point boundary value problem

$$y'' = f\left(x; y; y'\right);\ y(a) = \alpha;\ y(b) = \beta; \tag{2.33}$$

where $a < b$ and $x \in [a; b]$. Making an initial guess for $y(a)$ and denote by $y(xi)$ the solution of the initial value problem is

$$y'' = f\left(x; y; y'\right);\ y(a) = \alpha;\ y(b) = \beta. \tag{2.34}$$

Introducing the notation $Y\ (xi) = y(xi)$ and $\upsilon(x; 0) = \frac{\partial}{\partial x}\upsilon(x; 0)$equation (2.33) is rewritten as

$$\frac{\partial}{\partial x}Y(x; 0) = \upsilon(x; 0)\ ;\ \frac{\partial}{\partial x}\upsilon(x; 0) = f(x, Y(x; 0),\ \upsilon(x; 0)). \tag{2.35}$$

The solution Y (x$_i$) of the IVP (2.35) coincides with the solution $y(x)$ of the BVP (2.34) provided we can find a value of $Y\ (x)$ such that:

$$\varphi(x) \equiv Y(b_j) - \beta = 0 \tag{2.36}$$

The basic ideal of the shooting method for the numerical solution of the IVP of equation (2.24) is to find a root to the equation (2.26).

Here, the fourth order Runge-Kutta techniques was used to find the root and the scheme for the fourth order Runge-Kutta method is

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{2.37}$$

where, $k_1 = k_2 = k_3 = k_4 =$

The numerical code that incorporates the methods described above, using Maple 18 to solve the modelled equations was applied. Therefore, the models 1 and 2 were nonlinear coupled differential equations and their satisfying boundary conditions given in (2.32). The problems as BVP were solved by applying shooting technique (guessing the unknown values) to change the conditions BVPs. to IVP. Equations in the two models were integrated as IVPs, the values for $f''(0)$, $\theta'(0)$ and $\varphi'(0)$ which were required to explain skin friction, Nusselt and Sherwood numbers, but no such values were given at the boundary. The suitable guess values for $f''(0)$, $\theta'(0)$ and $\varphi'(0)$ were chosen and then integration was carried out. The researcher compared the calculated values for $f^{''}(0)$, $\theta^{'}(0)$ and $\varphi^{'}(0)$ at $\eta = 7$ with the given boundary conditions $f^{''}(7) = 0$, $\theta^{'}(7) = 0$ and $\varphi^{'}(7) = 0$. Then adjusted the estimated values for $f^{''}(0)$, $\theta^{'}(0)$ and $\varphi^{'}(0)$, to give a better approximation for the solution. The researcher performed series of computations to obtain values for $f^{''}(0)$, $\theta^{'}(0)$ and $\varphi^{'}(0)$, and then applied a fourth-order Runge-Kutta method with shooting technique with step-size $h = 0.01$. The above procedure was repeated until the results up to the desired degree of accuracy 10^{-6}. The numerical computation to obtain velocity, temperature and concentration profiles were presented. The effect of the computation on heat and free convective mass transfer as well as proportional effects on the skin friction coefficient, Nusselt number and local Sherwood number are presented and examined for different values of the independent parameters. These enabled the researcher to use the effects of dimensionless parameters to explain cross-diffusion on free convective heat and mass transfer of MHD flow in porousmedia. However, the physical quantities of practical interest are the local skin friction, theNusselt number Nu and local Sherwood number Sh defined respectively as:

$$C_f =, \; Nu =, \; Sh =$$

where k is the thermal conductivity of the fluid, τw, qw and qm are respectively given as

$$\tau_w =, \; q_w = \; q_m$$

Therefore, the local skin friction coefficient, local Nusselt number and local Sherwood are:

$$C_f Re_x^{\frac{1}{2}} = f^{''}(0), \; Nu Re_x^{-\frac{1}{2}} = \theta^{'}(0) \; Sh Re_x^{-\frac{1}{2}} = \phi^{'}(0)$$

where $Re_x^{\frac{1}{2}} = \frac{U_w x}{v}$ is the local Reynolds number and therefore, C_f, Nu and Sh were rewritten respectively as:

$$C_f \Big(\frac{U_w x}{v}^{\frac{1}{2}}\Big) = f^{''}(0), \; Nu\Big(\frac{U_w x}{v}^{-\frac{1}{2}}\Big) = \theta^{'}(0) \; Sh\Big(\frac{U_w x}{v}^{-\frac{1}{2}}\Big) = \phi^{'}(0)$$

3. Results and Discussion

Numerical values of $\theta^{'}(0)$ at the surface for different values of Pr and M and R when new parameters were zero.

Table 1 comparing the present study with the surface for $\theta^{'}(0)$.

Pr	M	R	Present study	[6]	[12]	[5]
1	0	1	0.56534	0.56319	0.5315	0.5315
1	2	1	0.43194	0.45053	0.4505	
1	1		0.87309	0.86139	3.6604	
1			0.95765	0.95743	0.9548	0.9548
2			1.47084	1.47898	1.4715	1.4714

Table 1 shows the variation of Pr. M. and R for numerical values of $\theta^{'}(0)$ when compared with the existing literature were in close agreement. The present study shows improvement over the previous studies. The empty cells in the table indicated independent parameters not considered (Bundi and Nazar, 2009). Numerical computation for $\theta^{'}(0)$ and $\Phi^{'}(0)$ at the surface for different values of Gr. Gc. M. Fw. and Sc when other parameters were made zero as in Table 2.

Table 2 comparing this study at the surface for $\theta'(0)$ and $\Phi'(0)$

Gr	Gc	M	Fw	Sc	Present study		[14]		[8].	
					$-\theta'(0)$	$-\Phi'(0)$	$-\theta'(0)$	$-\Phi'(0)$	$-\theta'(0)$	$-\Phi'(0)$
0.1	0.5	0.1	0.1	0.62	0.8436	0.7763	0.8385	0.7649	0.8421	0.7702
0.1	0.1	0.1	0.1	0.78	0.8038	0.8412	0.7973	0.8330	0.7937	0.8340
0.5	0.1	0.1	0.1	0.62	0.8410	0.7738	0.8359	0.7625	0.8379	0.7658
1.0	0.1	0.1	0.1	0.62	0.8762	0.8068	0.8730	0.7968	0.8753	0.8020

Table 2 compared the present and earlier studies at the surface. The variations of Gr, Gc, M, and fw for the numerical values of $-\theta'(0)$ and $-\Phi'(0)$ at the surface when compared with existing literature were found to be in close agreement. A higher percentage was noticed in the present study than the previous years. The effects of magnetic field parameter on the fluid flow, temperature and concentration provide. The increase in M showed that the rate of fluid had had resulted to the thinning of the velocity boundary layer. It was discovered that increased M caused respective increase in temperature and concentration layer get thicker as the magnetic parameter increased. Thermal Grashof Gr. effect showed that increased Gr led to increase in the flow rate as a result, the velocity boundary layer thickened. That is to say heat has strong influence on flow rate. The solutal Grashof Gc. effect showed increase in velocity profile, this was a result of free convection current, thereby increasing the velocity distribution. The effect of radiation on velocity and temperature showed increase as was as result of thickness in the momentum and thermal boundary layers. The effect of suction/ injection on the velocity, temperature and concentration profiles showed that suction caused decrease in velocity indicating stability at boundary layer while injection increased the velocity profile at the boundary. This indicated that injection supported flow to penetrate into the fluid. It must be noted that increased suction parameter decreased temperature profile.

This action was due to the fact that larger suction led to faster cooling of the plate. The temperature also increased as the injection parameter increased, this meant that heat transfer from the fluid to the surface. The figure showed that concentration decreased as suction parameter increased injection concentration due to respective thinness and thickness in the mass boundary layer. That showed that increase in Prandtl parameter decreased near the surface and temperature profile also decreased. This occurred because Pr. increases the thickness of the thermal boundary layer and heat was able to diffuse out of the system thereby decreasing the temperature profile.

Schmidt is defined as the ratio of momentum to the mass diffusivity. Increase in Sc. decreased velocity and concentration profiles. Sc. Number quantities the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic velocity and concentration layers. Chemical reaction effects on concentration showed decrease in the mass boundary layer thickness. The effects of heat source $\phi>0$ or heat sink ($\phi<0$) on temperature profile showed that heat source generated more energy thereby increasing the temperature of the fluid. The heat sink absorbed the energy, this thereby decreased temperature of the fluid. The plate or surface was inclined and when the Soret and Dufor effects were analysed along with the first model. The following independent variables of momentum equation, energy and species explained the behaviors of fluid flow, skin flow, temperature (Nusselt number) and concentration (Sherwood number.)

The second model showed the effects of increase in values of Gr, Gc, R, Q, Sc, λ thicken the thermal boundary layer by reducing the rate at which heat diffuses out of the system as well as chemical reaction. The increase in fw and Pr. Reduce the thickness of the thermal boundary layer. The effects of increased values of fw, Q, Sc. R and λ caused the thinning in concentration boundary layer while M, Da and Pr thicken the mass boundary layer. Increase in Dufor and Soret parameters increased the flow at the boundary layer. The significance of these cross diffusion parameters could not be totally neglected when explaining the concept of flow on vertical or inclined plates. All the parameters considered in the model 1 exhibited the same pattern of flow as model 2. They showed convergence at almost the same values.

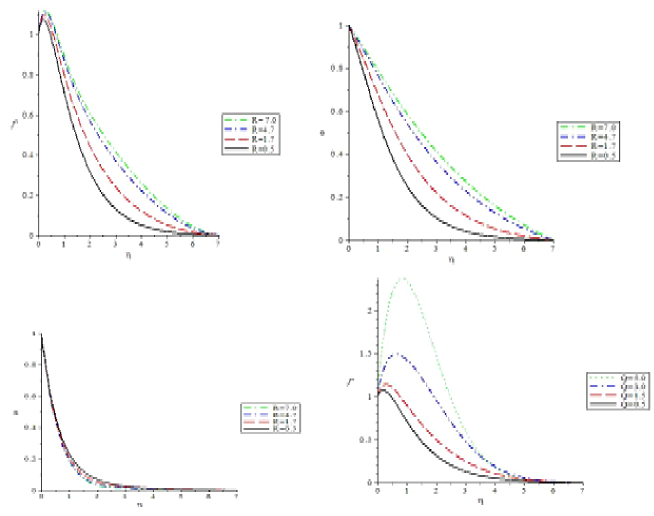

Figure 3:

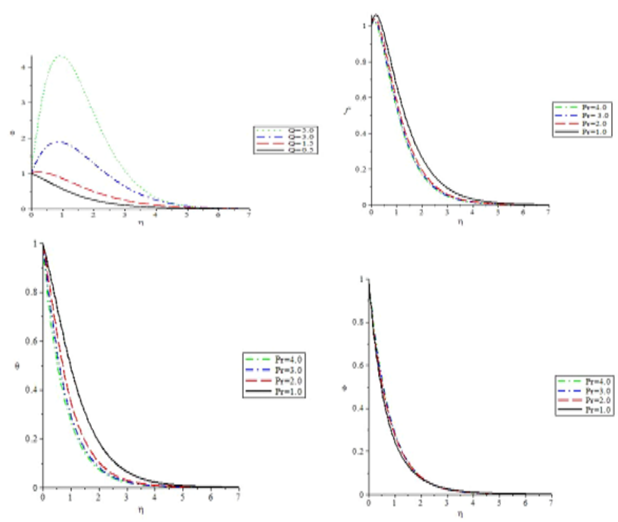

Figure 4:

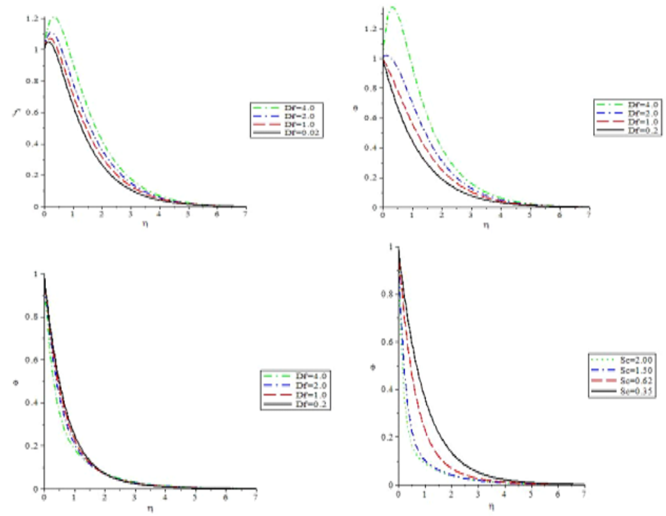

Figure 5:

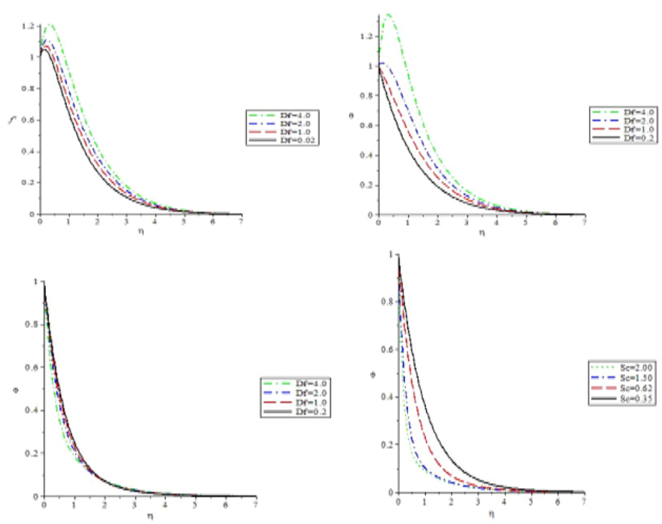

Figure 6:

Conclusions: This study analysed two models, the effect of thermal radiation in porous media was evaluated, the following independent parameters M, fw, $Q$and ? produced increase in radiation that led to increase in dependent variables, skin friction, Nusselt and Sherwood numbers but not in skin friction. The influence of fw and $Da\, of\, MHD$ fluid flow on vertically-stretching surface showed that increased magnetic parameter led to decreased skin friction, Nusselt and Sherwood numbers, Da, also increased the skin friction but decreased Nusselt and Sherwood numbers. The effects of $alpha$, $Dufor\ and\ Soret$ numbers showed increase in skin friction, Nusselt and Sherwood numbers. Radiation increased the skin friction and Sherwood but not the Nusselt number hence, the influence of cross diffusion parameters on heat and mass transfer of MHD fluid flow in porous media over exponentially stretching surface. The above graphs further explain the models.

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