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A Bootstrap Approach to Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA) Control Charts for Monitoring Process Means

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Abstract

This research proposed a bootstrapped method for setting control boundaries and detecting out-of-control signals. The proposed approaches for determining control limits are designed to address the issues of distributional assumptions and out-of-control signal detection. It enables the setting of control limits, which can improve the detection of the out control signal, according to the results of the performance evaluation. When compared to previous methods, bootstrapped methods improve the discovery of out of control signals in any process mean. The methods outperformed the conventional CUSUM, EWWMA and EWMA-CUSUM control chart in detection of early shift in a process mean.

1. Introduction

Statistical quality control is a well established discipline which is commonly applied in industry to monitor quality characteristics for an un-controllable situation [8]. The most popular method in the quality control is the statistical control charts. Since their introduction in 1947 by Walter Shewhart, these charts had

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been widely used to control process variation and to detect any change in the quality process [6]. For example, during World War II, Statistical Quality Control were employed with various modifications. On the other hand, Statistical Process Control (S.P.C), which is newer in particular, quality control techniques, such as control charts have become increasing important for the effective systematic monitoring of quality characteristics [10]. Statistical Control Charts are commonly employed to get stability in SPC. It also ensures products and process quality control [1].
Control Charts are one of the main tools of statistical process control. It is a graphical representation of a characteristic of the process under investigation. It is a tool used to identify special causes of variability in a process [4]. We usually have a target for each product we want to produce when we have a manufacturing process.
Regardless of how well we design the entire process or method, we are expected to be near to, but not always in, the target value [9]. Every process is affected by the presence of this variability. There are two kinds of variability: common cause (chance cause) variability and natural variability, which every process encounters. Randomness is the reason for its existence. From one product to the next, we can find entirely random variations [6]. A process that operates with only common cause of variability is said to be in control. The special cause (assignable cause) variability is a result of factors that are not purely random [7]. Those factors cause differences in the process and as a result, they affect the process leading to bad or low quality products [3]. When this happened, the process is said to be out-of-control. This type of variability can be detected with control chart, giving us the ability to remove its effect and therefore reduces the overall variability [11]. Removing special causes leads to the improvement of the quality of the products [5].

2. Research Methods

We intend to use four different methods to conduct this study: Cumulative Sum (CUSUM), Exponentially Weighted Moving Average (EWMA), joint EWMA-CUSUM and the proposed Bootstrap EWMA-CUSUM Control Chart method.

2.1. **Derivation of CUSUM Methods.** Let CUSUM control chart be defined as:

$$S_i = \sum_{i=1}^{n} X_i - \mu \tag{1}$$

where µ is the process mean and X_i is the sample data at time i
For increase in average the upper CUSUM is given as

$$S_i^+ = \max[0, \quad S_{i-1}^+ + (X_i - \mu) - k] \tag{2}$$

where k is the reference value
For the lower CUSUM,

(3) $$S_i^- = \min[0,\ S_{i-1}^- - (X_i - \mu) + k]$$

Decision:
Upper CUSUM: The chart detects signal if $S_i^+ > h$
Lower CUSUM: The chart detects signal if $S_i^- <$ -h
where h is the decision interval and k is the reference value

(4) $$k_1 = \mu + f\sigma = OutofControlMean$$

(5) $$k_2 = \mu - f\sigma = InControlMean$$

where f is the shift size

(6) $$k = \frac{k_2 - k_1}{logk_2 - logk_1}$$

2.2. **Joint EWMA-CUSUM (JEC) Control Chart.** We define EWMA control chart as:

(7) $$Z_i = \lambda x_i + (1 - \lambda)Z_{i-1}$$

where $Z_0 = 0$
Upper Control Limit

(8) $$UCL = \mu_0 + L\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]}$$

(9) $$LCL = \mu_0 - L\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]}$$

On standardizing the CUSUM on the basis of our proposed method, the standardized CUSUM becomes

(10) $$S_i = \frac{X_i - \mu}{\sigma}$$

where σ is the process standard deviation

(11) $$S_i^+ = \max[0,\quad \frac{X_i - \mu}{\sigma} + Z_{i-1} - \mu - k]$$

(12) $$S_i^+ = \min[0,\quad \frac{X_i - \mu}{\sigma} + Z_{i-1} - \mu + k]$$

Note that $JEC_0^+ = JEC_0^- = 0$

Let (10) be π, on substituting (8) into (9), we have

$$UCL = \mu + L\sigma\pi\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]} \tag{13}$$

$$LCL = \mu - L\sigma\pi\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]} \tag{14}$$

i = 1, 2, . . .
The joint EWMA/CUSUM chart becomes

$$UCL = \mu + L\sigma\left(\frac{X_i-\mu}{\sigma^2}\right)\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]} \tag{15}$$

$$LCL = \mu - L\sigma\left(\frac{X_i-\mu}{\sigma^2}\right)\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]} \tag{16}$$

$$UCL = \mu + L\left(\frac{X_i-\mu}{\sigma}\right)\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]} \tag{17}$$

$$LCL = \mu - L\left(\frac{X_i-\mu}{\sigma}\right)\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]} \tag{18}$$

2.3. **Bootstrap Algorithm for Cumulative Sum (CUSUM) Control Chart.**
Step 1: Assuming $x_1, x_2, x_3, \cdots, x_n$ are random variables taken from our production process with parameters μ and σ^2 known. If the population is not known, we estimate them [2].
Step 2: Compute the mean ($\overline{X}^*$) Variance (S^*) and the Standard Deviation (σ^*)
Step 3: For each sample i = 1,2,3, . . ., n, the following steps are followed:
(a) Compute Statistics C^+ and C^-
(b) Compute statistics $S_i{}^+$ and $S_i{}^-$
(c) Specify the reference value k
Step 4: Compute the bootstrap H
Step 5: Plot the statistics $C_i{}^+$ and $C_i{}^-$ against the number of sample.
Step 6: Compare statistics $C_i{}^+$ and $C_i{}^-$ with H. If any of the statistics $C_i{}^+$ and $C_i{}^-$ is greater than H, the shift will be identified accordingly.
(a) If $C_i{}^+ > H$, this indicates positive mean shift
(b) If C_i- $< H$, this also indicates negative mean shift

2.4. **Bootstrap Algorithm for EWMA Control Chart. Step 2:** Assuming $x_1, x_2, x_3, \cdots, x_n$ are random variables taken from our production process with parameters μ and σ^2 known. If the population is if unknown, it is estimated.
Step 2: Compute the (μ^*) and (σ^*) from the in control data
Step 3: Specify the smoothing parameter ?*
Step 4: Compute the Z* statistic for i observations from the in-control data set Using $Z_i^* = ?^* X_i^* + (1- ?)^* Z^*_{i-1}$, $Z_o^* = µ^*$
Step 5: Let Z_i^*, Z_2^*, . . ., Z_n^* be a set of Z* values from the Bootstrap sample (I = 1, ...B) drawn from a B bootstrap samples, $[Z_{\%lower}, Z_{\%upper}] = [N^{-1}{}_{(B)}, N^{-1}{}_{(1-B)}]$ where N is the cumulative distribution function of Z from the observed resample data and B ranges from 0 and 1
Step6: Use the control limit gotten to monitor a new observation. That is, BUCL $(B(1\text{-}\alpha)\frac{100}{0.95}$.

2.5. **Bootstrap Algorithm for Joint EWMA-CUSUM (JEC) Control Chart. Step 1 :** Predetermine the reference solve k*
Step 2: Specify the significant level B*, observation number n.
Step 3: Compute the standard deviation σ^* from the in - control data
Step 4: For N replications, follow the following steps
(a) Compute n samples which follows univariate distribution N (µ, σ^2) process
(b) Compute S_i^+ and S_i^- using
$(Y_i^+)^* = \max^*[0, \frac{Xi*-µ}{\sigma} + Z_{i-1}^*\text{-}µ^*\text{-}K^*]$
$(Y_i^-)^* = \min^*[0, \frac{Xi*-µ}{\sigma} + Z_{i-1}^*\text{-}µ^*\text{+}K^*]$
(c) Resample B Bootstrap samples from Y_i^*
(d) Arrange B bootstrap from the highest to the lowest (maximum to minimum)
Step 5: Compute the in-control ARL*. If the ARL* is not equal to the in-control ARL*
Step 6: Save the Control Limit with specified ARL*
Step 7: Repeat Step 1 with different values of k, significant level B, and the observation number n.

Step 8: compute Y-Statistics. That is:

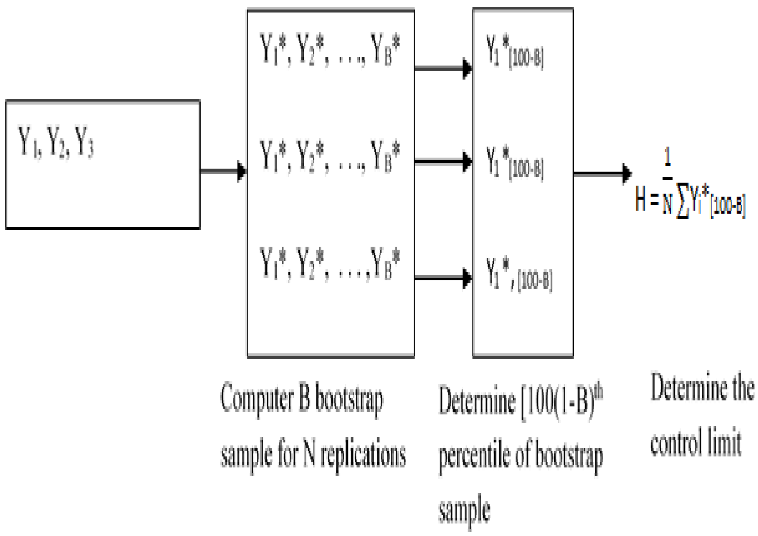

Figure 4.4: Monitoring Performance of JOINT EWMA-CUSUM Bootstrap for mean of wearer Heart rate data

Step 9: Obtain the Boostrap EWMA/CUSUM Statistics i.e Upper (h^{+}) and lower (h^{-}) control limits using step 7.

3. Results and Analysis

The analyses carried out were computing the control limits for the CUSUM, EWMA and JEC. The bootstrap approach was also computed for the three classical approaches. As usual, one method is said to be better than the other when it detects out of control for both upper and lower limits faster in a process mean. It is also shown numerically (in tabular form) that the proposed bootstrap approach enhanced the identification of out-of-control quality characteristics when compared with existing CUSUM, EWMA and JEC methods.

Table 1: The Wearer's Heart Rate Data (Irrua Specialist Teaching Hospital, Irrua, Edo State)

S/No. (i)	X_i	S/No. (i)	X_i	S/No. (i)	X_i
1.	79	51.	78	101.	79
2.	79	52.	81	102.	79
3.	81	53.	73	103.	83
4.	81	54.	79	104.	78
5.	84	55.	75	105.	85
6.	78	56.	80	106.	81
7.	81	57.	77	107.	84
8.	76	58.	82	108.	78
9.	80	59.	78	109	78
10.	82	60.	82	110.	77
11.	79	61.	81	111.	84
12.	77	62.	84	112.	80
13.	82	63.	82	113.	80
14.	81	64.	76	114	77
15.	79	65.	81	115	78
16.	79	66.	84	116	80
17.	81	67.	80	117	82
18.	81	68.	79	118	78
19.	76	69.	82	119	83
20.	81	70.	83	120	82
21.	79	71.	88	121	78
22.	78	72.	80	122	80
23.	78	73.	79	123	79
24.	79	74.	83	124	81
25.	86	75.	78	125	80
26.	80	76.	78	126	78
27.	75	77.	76	127	86
28.	80	78.	84	128	80
29.	81	79.	85	129	77
30.	79	80.	78	130	84
31.	79	81.	79	131	86
32.	73	82.	78	132	86
33.	78	83.	75	133	83
34.	81	84.	81	134	88
35.	81	85.	79	135	79

Table 1 Continues: The Wearer's Heart Rate Data (Irrua Specialist Teaching Hospital, Irrua, Edo State)

S/No. (i)	X_i	S/No. (i)	X_i	S/No. (i)	X_i
36.	80	86.	77	136	78
37.	80	87.	72	137	78
38.	82	88.	86	138	80
39.	78	89.	75	139	80
40.	84	90	78	140	86
41.	81	91.	79	141	77
42.	78	92.	81	142	79
43.	79	93.	80	143	80
44.	82	94.	78	144	82
45.	82	95.	78	145	78
46.	81	96.	84	146	79
47.	80	97.	78	147	78
48.	77	98.	76	148	80
49.	75	99.	79	149	76
50.	80	100.	80	150	83

3.1. **Monitoring Performance Evaluation of CUSUM for mean of wearer Heart rate data.** The tabular CUSUM was calculated using R software and is depicted in Table 2 and Figure 1 below.

Table 2: Monitoring Performance of CUSUM for mean of wearer Heart rate data

n	X_i	y_i	C_i^+	C_i^-	n	X_i	y_i	C_i^+	C_i^-
1	79.054	-0.2987	0	0	76	77.752	-0.7756	0.3179	0.5032
2	79.0229	-0.31	0	0	77	75.9826	-1.4238	0	1.427
3	81.2493	0.5055	0.0055	0	78	84.1692	1.5751	1.0751	0
4	80.918	0.3842	0	0	79	85.2459	1.9696	2.5447	0
5	84.3425	1.6386	1.1386	0	80	78.1243	-0.6392	1.4055	0.1392
6	77.6426	-0.8157	0	0.3157	81	78.9378	-0.3412	0.5642	0
7	80.7621	0.3271	0	0	82	77.7475	-0.7772	0	0.2772
8	75.6454	-1.5473	0	1.0473	83	74.7704	-1.8678	0	1.645
9	79.5382	-0.1213	0	0.6686	84	80.5923	0.2648	0	0.8802
10	81.9905	0.7771	0.2771	0	85	78.55	-0.4833	0	0.8635
11	78.7103	-0.4246	0	0	86	80.4058	0.1965	0	0.167
12	76.9228	-1.0794	0	0.5794	87	77.254	-0.958	0	0.625
13	82.2942	0.8883	0.3883	0	88	80.1596	0.1063	0	0.0187
14	81.3243	0.533	0.4213	0	89	74.9429	-1.8046	0	1.3233

Table 2 Continues: Monitoring Performance of CUSUM for mean of wearer Heart rate data

n	X_i	y_i	C_i^+	C_i^-	n	X_i	y_i	C_i^+	C_i^-
15	78.7447	-0.412	0	0	90	77.9926	-0.6875	0	1.5107
16	79.0059	-0.3163	0	0	91	79.0422	-0.303	0	1.3137
17	81.0095	0.4177	0	0	92	81.4986	0.5969	0.0969	0.2169
18	81.2386	0.5016	0.0016	0	93	80.0822	0.078	0	0
19	75.9739	-1.427	0	0.927	94	77.7704	-0.7689	0	0.2689
20	81.4935	0.595	0.095	0	95	78.1401	-0.6334	0	0.4023
21	78.7179	-0.4218	0	0	96	84.4383	1.6737	1.1737	0
22	78.2471	-0.5942	0	0.0942	97	77.5957	-0.8328	0	0.3328
23	78.0841	-0.654	0	0.2482	98	76.4805	-1.2414	0	1.0742
24	79.2471	-0.2279	0	0	99	78.6551	-0.4448	0	1.019
25	85.826	2.1821	1.6821	0	100	80.2771	0.1494	0	0.3697
26	80.8253	0.3502	1.5323	0	101	79.4464	-0.1549	0	0.0246
27	74.6523	-1.9111	0	1.4111	102	79.2688	-0.22	0	0
28	80.3826	0.188	0	0.723	103	82.5792	0.9927	0.4927	0
29	81.2023	0.4883	0	0	104	77.6889	-0.7987	0	0.2987
30	78.5817	-0.4717	0	0	105	84.8213	1.814	1.314	0
31	79.2056	-0.2431	0	0	106	80.6686	0.2928	1.1068	0
32	72.6384	-2.6488	0	**2.1488**	107	83.9838	1.5072	2.114	0
33	78.2372	-0.5979	0	**2.2467**	108	77.778	-0.7661	0.848	0.2661
34	80.6133	0.2725	0	1.4741	109	78.2699	-0.5859	0	0.352
35	81.2753	0.5151	0.0151	0.4591	110	76.6562	-1.177	0	1.029
36	79.9715	0.0374	0	0	111	83.6795	1.3957	0.8957	0
37	80.1232	0.093	0	0	112	80.0128	0.0526	0.4483	0
38	82.1	0.8171	0.3171	0	113	80.0491	0.0659	0.0142	0
39	78.2953	-0.5766	0	0.0766	114	76.6996	-1.1611	0	0.6611
40	84.1144	1.5551	1.0551	0	115	78.3646	-0.5512	0	0.7123
41	80.5123	0.2356	0.7906	0	116	79.6936	-0.0644	0	0.2767
42	77.9807	-0.6918	0	0.1918	117	82.1301	0.8282	0.3282	0
43	79.4495	-0.1538	0	0	118	78.2457	-0.5947	0	0.0947
44	82.2453	0.8704	0.3704	0	119	83.1505	1.202	0.702	0
45	81.5761	0.6252	0.4956	0	120	82.2626	0.8767	1.0787	0
46	81.0136	0.4192	0.4148	0	121	78.1264	-0.6384	0	0.1384
47	79.7373	-0.0484	0	0	122	79.8429	-0.0097	0	0
48	76.6667	-1.1732	0	0.6732	123	79.216	-0.2393	0	0
49	74.8005	-1.8568	0	2.0299	124	80.9019	0.3783	0	0
50	79.8045	-0.0237	0	1.5537	125	79.9528	0.0306	0	0

Table 2 Continues: Monitoring Performance of CUSUM for mean of wearer Heart rate data

n	X_i	y_i	C_i^+	C_i^-	n	X_i	y_i	C_i^+	C_i^-
51	77.9728	-0.6947	0	1.7484	126	78.1849	-0.617	0	0.1170
52	81.4265	0.5704	0.0704	0.678	127	86.311	2.3597	1.8597	0
53	73.462	-2.3471	0	**2.5251**	128	80.1708	0.1104	1.4701	0
54	78.7516	-0.4094	0	**2.4345**	129	76.8996	-1.0879	0	0.5879
55	75.039	-1.7694	0	**3.7039**	130	83.7783	1.4319	0.9319	0
56	80.4863	0.226	0	**2.9779**	131	85.812	2.1769	2.6089	0
57	77.454	-0.8848	0	**3.3627**	132	86.1862	2.314	**4.4229**	0
58	81.5027	0.5984	0.0984	**2.2643**	133	82.9571	1.1311	**5.054**	0
59	77.8226	-0.7498	0	**2.5141**	134	79.779	-0.0331	**4.5209**	0
60	81.7732	0.6974	0.1974	1.3167	135	78.9599	-0.3331	**3.6878**	0
61	80.9166	0.3836	0.0811	0.433	136	77.5782	-0.8393	2.3485	0.3393
62	84.1186	1.5566	1.1377	0	137	78.4456	-0.5215	1.327	0.3608
63	82.3542	0.9103	1.5479	0	138	79.9232	0.0197	0.8468	0
64	76.1682	-1.3558	0	0.8558	139	80.0245	0.0568	0.4036	0
65	80.8873	0.3729	0	0	140	86.1716	2.3087	2.2123	0
66	83.7078	1.4061	0.9061	0	141	76.847	-1.1071	0.6051	0.6071
67	79.718	-0.0554	0.3507	0	142	78.9305	-0.3439	0	0.451
68	79.1786	-0.253	0	0	143	79.7257	-0.0526	0	0.0036
69	81.8383	0.7213	0.2213	0	144	82.0611	0.8029	0.3029	0
70	82.7864	1.0686	0.7899	0	145	77.7296	-0.7838	0	0.2838
71	87.9164	2.9478	**3.2377**	0	146	78.8698	-0.3661	0	0.1499
72	80.2083	0.1242	**2.8619**	0	147	77.9345	-0.7087	0	0.3587
73	78.899	-0.3554	2.0064	0	148	79.7974	-0.0263	0	0
74	83.4582	1.3147	**2.8211**	0	149	75.5399	-1.586	0	1.086
75	77.883	-0.7276	1.5935	0.2276	150	82.9402	1.1249	0.6249	0

mean = 79.8693, SD = 2.7299, h =5.09, Bh^+ = 2.5583, Bh^- = 2.0987, where Bh^+ = Bootstrap Upper Control Limit, Bh^- = Bootstrap Lower Control Limit, SD = Standard Division and H = Decision interval

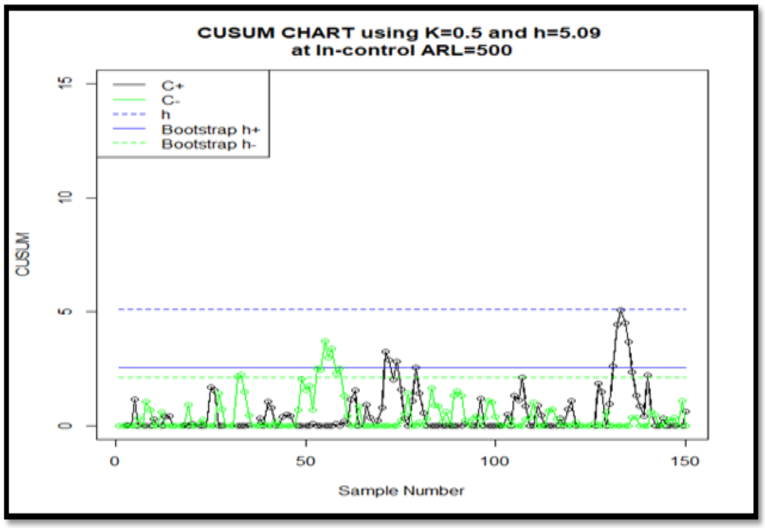

Figure 1: Monitoring Performance of CUSUM for Wearer Heart rate data

From figure 1, the bootstrap CUSUM chart detected about 9 points before signal is made at upper CUSUM level, while at lower CUSUM level; about 4 points were out of control. Again, the classical CUSUM statistics is unable to detect changes in the process mean. Therefore, the bootstrapped CUSUM detected shift in the process mean faster than the classical CUSUM. This was made possible as we translated Algorithm 1 to univariate bootstrap control system using R codes. Bootstrap sample were replicated 5000 times starting with the initial set of observation and CUSUM value computed for each sample as showed in Table 2. The control limits were determined to be 2.5583 for the upper CUSUM and 2.0987 for the lower limits. And those values that were above and below the boundaries were underlined.

Discussion and Interpretation of Out of Control Limits in Table 2

From Table 2, it was observed that sample 53 - 59 and 132 – 135 were out of control as underlined. This shows that our proposed method base on Algorithm 1 and R code perform better in setting control limits and identifying out of control signals over the existing classical method which doesn't detect any signal.

Table 3: Monitoring Performance of EWMA for mean of wearer Heart rate data.

n	X_i	y_i	$EWMA_i$	n	X_i	y_i	$EWMA_i$
1	79.054	-0.2987	-0.0747	76	77.752	-0.7756	0.0908
2	79.0229	-0.31	-0.1335	77	75.9826	-1.42379	-0.288
3	81.2493	0.50554	0.02625	78	84.1692	1.575128	0.1779
4	80.918	0.38417	0.11573	79	85.2459	1.969566	0.6258
5	84.3425	1.63862	0.49645	80	78.1243	-0.63923	0.3096
6	77.6426	-0.8157	0.16842	81	78.9378	-0.34124	0.1469
7	80.7621	0.32706	0.20808	82	77.7475	-0.77724	-0.084
8	75.6454	-1.5473	-0.2308	83	74.7704	-1.86781	-0.53
9	79.5382	-0.1213	-0.2034	84	80.5923	0.264834	-0.331
10	81.9905	0.77705	0.04171	85	78.55	-0.48327	-0.369
11	78.7103	-0.4246	-0.0749	86	80.4058	0.196523	-0.228
12	76.9228	-1.0794	-0.326	87	77.254	-0.95805	-0.41
13	82.2942	0.8883	-0.0224	88	80.1596	0.10635	-0.281
14	81.3243	0.533	0.11644	89	74.9429	-1.80463	-0.662
15	78.7447	-0.412	-0.0157	90	77.9926	-0.68745	**-0.668**
16	79.0059	-0.3163	-0.0908	91	79.0422	-0.303	-0.577
17	81.0095	0.41766	0.0363	92	81.4986	0.596851	-0.284
18	81.2386	0.50162	0.15263	93	80.0822	0.078008	-0.193
19	75.9739	-1.427	-0.2423	94	77.7704	-0.76887	-0.337
20	81.4935	0.59499	-0.033	95	78.1401	-0.63344	-0.411
21	78.7179	-0.4218	-0.1302	96	84.4383	1.673716	0.11
22	78.2471	-0.5942	-0.2462	97	77.5958	-0.83284	-0.126
23	78.0841	-0.654	-0.3481	98	76.4805	-1.2414	-0.405
24	79.2471	-0.2279	-0.3181	99	78.6551	-0.4448	-0.415
25	85.826	2.18205	0.30696	100	80.2771	0.149381	-0.274
26	80.8253	0.35022	0.31777	101	79.4464	-0.15492	-0.244
27	74.6523	-1.9111	-0.2394	102	79.2688	-0.21997	-0.238
28	80.3826	0.18804	-0.1326	103	82.5792	0.992707	0.0697
29	81.2023	0.48832	0.02265	104	77.6889	-0.7987	-0.147
30	78.5817	-0.4717	-0.1009	105	84.8213	1.814012	0.343
31	79.2056	-0.2431	-0.1365	106	80.6686	0.292806	0.3304
32	72.6384	-2.6488	**-0.7646**	107	83.9838	1.507231	0.6246
33	78.2372	-0.5979	**-0.7229**	108	77.778	-0.76609	0.2769
34	80.6133	0.27253	-0.474	109	78.2699	-0.58587	0.0612
35	81.2753	0.51506	-0.2268	110	76.6562	-1.17703	-0.248
36	79.9715	0.03744	-0.1607	111	83.6795	1.395742	0.1627
37	80.1232	0.09301	-0.0973	112	80.0128	0.052563	0.1352
38	82.1	0.81715	0.13133	113	80.0491	0.065857	0.1178
39	78.2953	-0.5766	-0.0456	114	76.6996	-1.16113	-0.202
40	84.1144	1.55508	0.35453	115	78.3646	-0.55119	-0.289

Table 3 Continues: Monitoring Performance of EWMA for mean of wearer Heart rate data.

n	X_i	y_i	$EWMA_i$	n	X_i	y_i	$EWMA_i$
41	80.5124	0.23557	0.32479	116	79.6936	-0.06435	-0.233
42	77.9807	-0.6918	0.07064	117	82.1301	0.828182	0.0323
43	79.4495	-0.1538	0.01454	118	78.2457	-0.59474	-0.124
44	82.2453	0.87037	0.2285	119	83.1506	1.201988	0.2071
45	81.5761	0.62524	0.32768	120	82.2627	0.876734	0.3745
46	81.0136	0.41919	0.35056	121	78.1264	-0.63844	0.1213
47	79.7373	-0.0484	0.25083	122	79.8429	-0.00968	0.0886
48	76.6667	-1.1732	-0.1052	123	79.2161	-0.23929	0.0066
49	74.8005	-1.8568	-0.5431	124	80.9019	0.378261	0.0995
50	79.8045	-0.0237	-0.4132	125	79.9528	0.03058	0.0823
51	77.9728	-0.6947	-0.4836	126	78.1849	-0.61702	-0.093
52	81.4265	0.57043	-0.2201	127	86.311	2.359703	0.5205
53	73.462	-2.3471	**-0.7518**	128	80.1708	0.110438	0.418
54	78.7516	-0.4094	**-0.6662**	129	76.8996	-1.08786	0.0415
55	75.039	-1.7694	**-0.942**	130	83.7783	1.431938	0.3891
56	80.4863	0.22603	-0.65	131	85.8121	2.176947	**0.8361**
57	77.454	-0.8848	**-0.7087**	132	86.1862	2.31399	**1.2056**
58	81.5027	0.59835	-0.3819	133	82.9571	1.131135	**1.187**
59	77.8226	-0.7498	-0.4739	134	79.779	-0.03307	**0.8819**
60	81.7732	0.69744	-0.1811	135	78.9599	-0.33313	0.5782
61	80.9166	0.38364	-0.0399	136	77.5782	-0.83927	0.2238
62	84.1186	1.5566	0.35923	137	78.4456	-0.52153	0.0375
63	82.3542	0.91026	0.49699	138	79.9232	0.019748	0.033
64	76.1682	-1.3558	0.0338	139	80.0245	0.056849	0.039
65	80.8873	0.3729	0.11857	140	86.1716	2.308664	0.6064
66	83.7078	1.40611	0.44046	141	76.847	-1.10714	0.178
67	79.718	-0.0554	0.31649	142	78.9305	-0.3439	0.0475
68	79.1786	-0.253	0.17411	143	79.7257	-0.05259	0.0225
69	81.8383	0.72129	0.31091	144	82.0611	0.802917	0.2176
70	82.7864	1.06859	0.50033	145	77.7296	-0.78379	-0.033
71	87.9164	2.94781	**1.1122**	146	78.8698	-0.36614	-0.116
72	80.2083	0.12418	**0.86519**	147	77.9345	-0.70875	-0.264
73	78.899	-0.3554	0.56004	148	79.7974	-0.02634	-0.205
74	83.4582	1.31469	**0.7487**	149	75.5399	-1.58595	-0.55
75	77.883	-0.7276	0.37962	150	82.9402	1.124915	-0.131

mean = 79.8693, SD=2.7299, UCL/LCL = ± 1.1339, BUCL/BLCL = ± 0.6672.

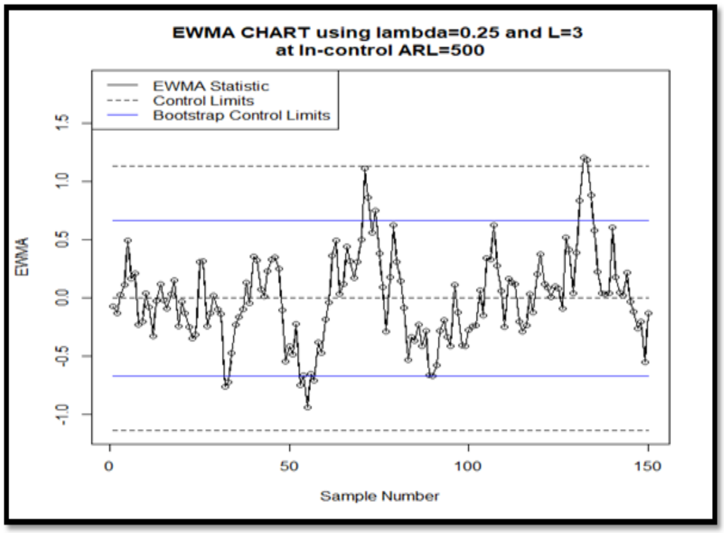

Figure 2: Monitoring Performance of EWMA for Wearer Heart rate data

From figure 2, the bootstrap EWMA control limits are lower than the Classical EWMA statistics. Both the upper and lower control limits of the bootstrap EWMA detected about 7 points above and below the control limits, whereas the Classical EWMA statistics was only able to detect about 2 shifts before signal.

Discussion and Interpretation of out of Control Results for EWMA
From the Table 3, it was observed that the classical EWMA detected two points than the out of control. However, the bootstrap EWMA was in addition able to detect shift in observations 32-33, 53-55, 57, 71-71, 74 at the upper level and 90, 131-134 at the lower level respectively. This was made possible with the translation of Algorithm 2 into R code.

Table 4: Monitoring Performance of Joint EWMA/CUSUM Bootstrap for mean of wearer Heart rate data

n	X_i	y_i	Q_i	EC_i^+	EC_i^-	n	X_i	y_i	Q_i	EC_i^+	EC_i^-
1	79.054	-0.299	-0.075	0	0	76	77.75	-0.776	0.091	3.673	0
2	79.023	-0.31	-0.134	0	0	77	75.98	-1.424	-0.288	3.196	0.099
3	81.249	0.506	0.026	0	0	78	84.17	1.575	0.178	3.185	0
4	80.918	0.384	0.116	0	0	79	85.25	1.97	0.626	3.622	0
5	84.343	1.639	0.496	0.307	0	80	78.12	-0.639	0.31	3.742	0
6	77.643	-0.816	0.168	0.287	0	81	78.94	-0.341	0.147	3.7	0
7	80.762	0.327	0.208	0.306	0	82	77.75	-0.777	-0.084	3.427	0
8	75.645	-1.547	-0.231	0	0.042	83	74.77	-1.868	-0.53	2.708	0.341

Table 4 Continues: Monitoring Performance of Joint EWMA/CUSUM Bootstrap for mean of wearer Heart rate data

n	X_i	y_i	Q_i	EC_i^+	EC_i^-	n	X_i	y_i	Q_i	EC_i^+	EC_i^-
9	79.538	-0.121	-0.203	0	0.056	84	80.59	0.265	-0.331	2.188	0.483
10	81.991	0.777	0.042	0	0	85	78.55	-0.483	-0.369	1.629	0.664
11	78.71	-0.425	-0.075	0	0	86	80.41	0.197	-0.228	1.212	0.703
12	76.923	-1.079	-0.326	0	0.137	87	77.25	-0.958	-0.41	0.613	0.924
13	82.294	0.888	-0.022	0	0	88	80.16	0.106	-0.281	0.143	1.016
14	81.324	0.533	0.116	0	0	89	74.94	-1.805	-0.662	0	1.489
15	78.745	-0.412	-0.016	0	0	90	77.99	-0.687	-0.668	0	1.969
16	79.006	-0.316	-0.091	0	0	91	79.04	-0.303	-0.577	0	2.357
17	81.009	0.418	0.036	0	0	92	81.5	0.597	-0.284	0	2.452
18	81.239	0.502	0.153	0	0	93	80.08	0.078	-0.193	0	2.456
19	75.974	-1.427	-0.242	0	0.053	94	77.77	-0.769	-0.337	0	2.604
20	81.494	0.595	-0.033	0	0	95	78.14	-0.633	-0.411	0	2.826
21	78.718	-0.422	-0.13	0	0	96	84.44	1.674	0.11	0	2.527
22	78.247	-0.594	-0.246	0	0.057	97	77.6	-0.833	-0.126	0	2.464
23	78.084	-0.654	-0.348	0	0.216	98	76.48	-1.241	-0.405	0	2.679
24	79.247	-0.228	-0.318	0	0.345	99	78.66	-0.445	-0.415	0	2.905
25	85.826	2.182	0.307	0.118	0	100	80.28	0.149	-0.274	0	2.99
26	80.825	0.35	0.318	0.247	0	101	79.45	-0.155	-0.244	0	3.045
27	74.652	-1.911	-0.239	0	0.05	102	79.27	-0.22	-0.238	0	3.094
28	80.383	0.188	-0.133	0	0	103	82.58	0.993	0.07	0	2.835
29	81.202	0.488	0.023	0	0	104	77.69	-0.799	-0.147	0	2.793
30	78.582	-0.472	-0.101	0	0	105	84.82	1.814	0.343	0.154	2.262
31	79.206	-0.243	-0.136	0	0	106	80.67	0.293	0.33	0.295	1.742
32	72.638	-2.649	-0.765	0	0.576	107	83.98	1.507	0.625	0.731	0.929
33	78.237	-0.598	-0.723	0	1.109	108	77.78	-0.766	0.277	0.819	0.463
34	80.613	0.273	-0.474	0	1.395	109	78.27	-0.586	0.061	0.691	0.212
35	81.275	0.515	-0.227	0	1.432	110	76.66	-1.177	-0.248	0.254	0.272
36	79.972	0.037	-0.161	0	1.404	111	83.68	1.396	0.163	0.228	0
37	80.123	0.093	-0.097	0	1.312	112	80.01	0.053	0.135	0.174	0
38	82.1	0.817	0.131	0	0.992	113	80.05	0.066	0.118	0.103	0
39	78.295	-0.577	-0.046	0	0.849	114	76.7	-1.161	-0.202	0	0.013
40	84.114	1.555	0.355	0.166	0.305	115	78.37	-0.551	-0.289	0	0.113
41	80.512	0.236	0.325	0.301	0	116	79.69	-0.064	-0.233	0	0.157
42	77.981	-0.692	0.071	0.183	0	117	82.13	0.828	0.032	0	0
43	79.45	-0.154	0.015	0.009	0	118	78.25	-0.595	-0.124	0	0
44	82.245	0.87	0.228	0.048	0	119	83.15	1.202	0.207	0.018	0

Table 4 Continues: Monitoring Performance of Joint EWMA/CUSUM Bootstrap for mean of wearer Heart rate data

n	X_i	y_i	Q_i	EC_i^+	EC_i^-	n	X_i	y_i	Q_i	EC_i^+	EC_i^-
45	81.576	0.625	0.328	0.187	0	120	82.26	0.877	0.375	0.204	0
46	81.014	0.419	0.351	0.348	0	121	78.13	-0.638	0.121	0.136	0
47	79.737	-0.048	0.251	0.41	0	122	79.84	-0.01	0.089	0.036	0
48	76.667	-1.173	-0.105	0.116	0	123	79.22	-0.239	0.007	0	0
49	74.801	-1.857	-0.543	0	0.354	124	80.9	0.378	0.1	0	0
50	79.804	-0.024	-0.413	0	0.578	125	79.95	0.031	0.082	0	0
51	77.973	-0.695	-0.484	0	0.873	126	78.19	-0.617	-0.093	0	0
52	81.426	0.57	-0.22	0	0.904	127	86.31	2.36	0.521	0.332	0
53	73.462	-2.347	-0.752	0	1.467	128	80.17	0.11	0.418	0.561	0
54	78.752	-0.409	-0.666	0	1.944	129	76.9	-1.088	0.042	0.413	0
55	75.039	-1.769	-0.942	0	2.697	130	83.78	1.432	0.389	0.613	0
56	80.486	0.226	-0.65	0	**3.158**	131	85.81	2.177	0.836	1.26	0
57	77.454	-0.885	-0.709	0	**3.678**	132	86.19	2.314	1.206	2.277	0
58	81.503	0.598	-0.382	0	**3.871**	133	82.96	1.131	1.187	3.275	0
59	77.823	-0.75	-0.474	0	**4.156**	134	79.78	-0.033	0.882	3.968	0
60	81.773	0.697	-0.181	0	**4.148**	135	78.96	-0.333	0.578	**4.357**	0
61	80.917	0.384	-0.04	0	**3.999**	136	77.58	-0.839	0.224	**4.392**	0
62	84.119	1.557	0.359	0.17	**3.451**	137	78.45	-0.522	0.037	**4.24**	0
63	82.354	0.91	0.497	0.478	2.765	138	79.92	0.02	0.033	**4.084**	0
64	76.168	-1.356	0.034	0.323	2.542	139	80.02	0.057	0.039	3.934	0
65	80.887	0.373	0.119	0.253	2.234	140	86.17	2.309	0.606	**4.352**	0
66	83.708	1.406	0.44	0.504	1.605	141	76.85	-1.107	0.178	**4.341**	0
67	79.718	-0.055	0.316	0.632	1.099	142	78.93	-0.344	0.048	**4.2**	0
68	79.179	-0.253	0.174	0.617	0.736	143	79.73	-0.053	0.023	**4.033**	0
69	81.838	0.721	0.311	0.739	0.237	144	82.06	0.803	0.218	**4.062**	0
70	82.786	1.069	0.5	1.05	0	145	77.73	-0.784	-0.033	3.84	0
71	87.916	2.948	1.112	1.973	0	146	78.87	-0.366	-0.116	3.535	0
72	80.208	0.124	0.865	2.649	0	147	77.94	-0.709	-0.264	3.082	0.075
73	78.899	-0.355	0.56	3.021	0	148	79.8	-0.026	-0.205	2.688	0.091
74	83.458	1.315	0.749	3.58	0	149	75.54	-1.586	-0.55	1.949	0.452
75	77.883	-0.728	0.38	3.771	0	150	82.94	1.125	-0.131	1.629	0.394

mean = 79.8693, SD = 2.7299, H=7.627, $BH^+ = 4.0248$, $BH^- = 3.1224$.

From Table 4 above, it was observed that joint EWMA-CUSUM analysis showed that bootstrap was out of control at observation 56-62 and observations 135-144 at the upper level which was not detected by the classical EWMA/CUSUM Chart.

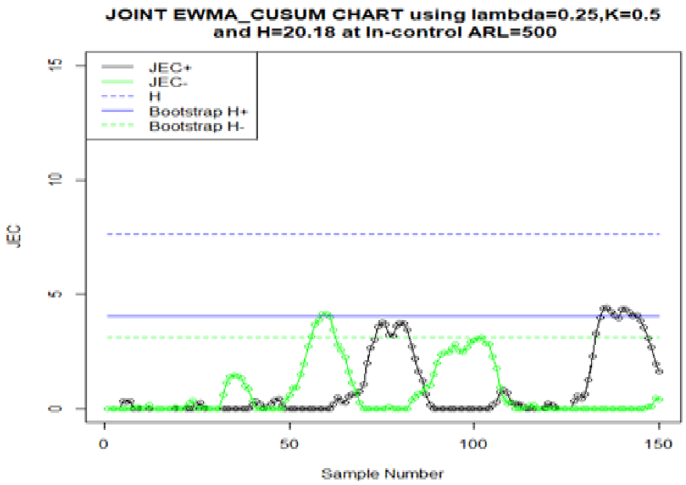

Figure 3: Monitoring Performance of JOINT EWMA-CUSUM Bootstrap for mean of wearer Heart rate data

From Figure 3 above, the Joint Bootstrap EWMA-CUSUM chart is more sensitive to changes in the process mean than the classical joint EWMA-CUSUM.

Conclusions: Non parametric approaches such as bootstrap were used to determine CUSUM, EWMA, and joint CUSUM-EWMA control limits for detecting small shifts in the system mean vector. The bootstrap control limits findings were compared to the results of the standard CUSUM, EWMA, and combination EWMA/CUSUM methods. According to the findings, the bootstrap method makes setting up control boundaries and detecting out of control signals faster and more reliable than traditional methods.

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Appendix 1: R Codes for Plotting CUSUM Control Chart

```
CUSUM=function(k,h,sig, mu, n,S){
shift= c(0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.5, 0.75, 1, 1.5, 2,3,4,5)
y= c() ; rl=matrix(0,S,length(shift));arl=c();sdt=c(); mup=mu; sd = sqrt((sig∧2/n))
for(d in 1:length(shift)) {
rlm=c()
for(j in 1:S)
{
m= mu + shift[d]*(sig/sqrt(n))
Cp = Cn =c()
for(i in 1:1000)
{
x = rnorm(n, m, sig); y = mean(x)
y= (y-mup)/sd
if(i==1)
{Cp[i] = max(0,(y-0) - k + 0)
Cn[i] = max(0,-(y-0) - k + 0)}
else
{Cp[i] = max(0,(y-0) - k + Cp[i-1])
Cn[i] = max(0,-(y-0) - k + Cp[i-1])}
if(Cp[i] > h || Cn[i]>h)
{rlm[j]=i; break}
else
{rlm[j]=0}
}
}
rl[,d]= rlm
arl[d] = mean( rlm)
sdt[d]=sd( rlm)
}
x1=matrix(round(cbind(arl,sdt),2),length(shift),2)
colnames(x1)=c(”ARL”, ”SDARL”)
rownames(x1)=shift
y1= rl
list(x1,y1)
}
library(spc)
h3=xcusum.crit(0.5,500,0,side=”two”)
CUSUM0_5=CUSUM(k=0.5,h=h3,sig=1, mu=0, n=n1, S=50000)[[1]]
CUSUM0_5
```

Appendix 2: R Codes for Plotting EWMA Control Chart

```
EWMA = function(K,H,k,h,ld,sig,mu,n,S){
shift= c(0,0.05,0.1, 0.15, 0.2, 0.25, 0.5,0.75,1,1.5,2,3,4,5);K=c();H=c()
y= c() ; rlm=matrix(0,S,length(shift));arl=c();sdt=c();sdp=c()
muy=mu; sdy = sqrt((sig^2/n))
for(d in 1:length(shift)){
rln=c()
for(j in 1:S){
m= mu + shift[b]*(sig/sqrt(n))
M1=M2=c();Z=c();
for(i in 1:1000000){
y= rnorm(n, m, sig)
if(i==1)
{Z[i]=ld*y+(1-ld)*mu;}
else{Z[i]=ld*y+(1-ld)*Z[i-1];}
sdp[i]=sdy*sqrt((ld/(2-ld))*(1-(1-ld)^(2*i)))
K[i]=k*sdp[i]
H[i]=h*sdp[i]
if(i==1)
{M1[i]=max(0,Z[i]-K[i]);}
else{M1[i]=max(0,Z[i]-K[i]+M1[i-1]);}
if(i==1)
{M2[i]=max(0,-Z[i]-K[i]);}
else{M2[i]=max(0,-Z[i]-K[i]+M2[i-1]);}
resamples1 <- lapply(1:1000, function(i) sample(M1, replace = T))
X1<-(sapply(resamples,quantile,0.95))
H1=mean(X1)
resamples<- lapply(1:1000, function(i) sample(M2, replace = T))
X2<-(sapply(resamples2,quantile,0.95))
H2=mean(X2)
if(M1[i]>H1| M2[i]>H2)
{rln[j]=i; break;}
else{rln[j]=0;}
}
}
rlm[,d]= rln
arl[d] = mean(rln)
sdt[d]=sd(rln)
}
x_1=matrix(round(cbind(arl,sdt),4),length(shift),2)
```

```
colnames(x_1)=c("ARL", "SDARL")
rownames(x_1)=shift
y_1= rlm
list(x_1,y_1)
}
EWMA_CUSUM01=EWMACUSUM(K=K,H=H,k=0.5,h=37.42,ld=0.1,sig=1, mu=0,
n=1, S=50000)[[1]]
EWMA_CUSUM01
```

Appendix 3: R Codes for Plotting EWMA-CUSUM Control Chart

```
set.seed(100)
library(spc)
EWMACUSUM=function(K,H,H1,H2,k,h,ld,sig,mu,n,S){
shift= c(0,0.05,0.1, 0.15, 0.2, 0.25, 0.5,0.75,1,1.5,2,3,4,5);K=c();H=c()
y= c() ; rlm=matrix(0,S,length(shift));arl=c();sdt=c();sdp=c()
muy=mu; sdy = sqrt((sig∧2/n))
for(d in 1:length(shift)){
rln=c()
for(j in 1:S){
m= mu + shift[d]*(sig/sqrt(n))
M1=M2=c();Z=c();
for(i in 1:1000000){
y= rnorm(n, m, sig)
if(i==1)
{Z[i]=ld*y+(1-ld)*mu;}
else{Z[i]=ld*y+(1-ld)*Z[i-1];}
sdp[i]=sdy*sqrt((ld/(2-ld))*(1-(1-ld)∧(2*i)))
K[i]=k*sdp[i]
H[i]=h*sdp[i]
if(i==1)
{M1[i]=max(0,Z[i]-K[i]);}
else{M1[i]=max(0,Z[i]-K[i]+M1[i-1]);}
if(i==1)
{M2[i]=max(0,-Z[i]-K[i]);}
else{M2[i]=max(0,-Z[i]-K[i]+M2[i-1]);}
resamples1 <- lapply(1:1000, function(i) sample(M1, replace = T))
X1<-(sapply(resamples1,quantile,0.95))
H1=mean(X1)
resamples2 <- lapply(1:1000, function(i) sample(M2, replace = T))
```

```
X2<-(sapply(resamples2,quantile,0.95))
H2=mean(X2)
if(M1[i]>H1| M2[i]>H2)
{rln[j]=i; break;}
else{rln[j]=0;}
}
}
rlm[,d]= rln
arl[d] = mean(rln)
sdt[d]=sd(rln)
}
x_1=matrix(round(cbind(arl,sdt),4),length(shift),2)
colnames(x_1)=c("ARL", "SDARL")
rownames(x_1)=shift
y_1= rlm
list(x_1,y_1)
}
EWMA_CUSUM01=EWMACUSUM(K=K,H1=H1,H2=H2,k=0.5,h=37.42,ld=0.1,sig=1,
mu=0, n=1, S=50000)[[1]]
EWMA_CUSUM01
set.seed(100)
library(spc)
```