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Heat and Mass Transfer on Mhd Oscillatory Flow of a Couple Stress Fluid in an Asymmetric Tapered Channel with Suction Effects

Abubakar Magaji*, Joseph Kpop Moses and Isah Jibril

Abstract

Heat and Mass Transfer on MHD Oscillatory Flow of a Couple Stress Fluid in Asymmetric Tapered Channel with suction effects. The temperatures prescribed at the channel walls are non-uniform. Magnetic field strength which is uniform is applied transversely to the channel. Dimensionless parameters are used to non-dimensionalized the governing equations to dimensionless form. Closed form solution method is used to solve the dimensionless equations govern the flow and the solutions for velocity, temperature and concentration distribution are obtained. The effects of flow parameters on velocity profile, temperature distribution amd concentration distribution are presented, discussed and shown graphically in details. The results of this work is in agreement with the results obtained in [11] when the suction parameter $s = 0$. It can be concluded that Increase in suction parameter s accelerates the velocity of the fluid. Increase in suction parameter s declines the temperature distribution. Higher values of Hartmann number M retards fluid velocity. Increase in Peclet number decreases the temperature distribution. And increase in radiation parameter N increases the temperature distribution.

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Department of Mathematical Sciences, Kaduna State University, Kaduna, Nigeria
E-mail of the corresponding author: abu_magaji@kasu.edu.ng
ORCID of the corresponding author: 0000-0002-6382-4619

1. Introduction

Heat transfer in the other hand can be said to be the transmission of energy from one region to another as a result of increase in temperature difference between them. While mass transfer can also be said to be the transportation or movement of materials or substances from a region of higher concentration to a region of lower concentration through a semi permeable membrane. Heat transfer occurs when there is temperature difference between two or more regions or surroundings. Much of the understanding of plasma came from the study of Magneto hydrodynamics (MHD) as it is the study of interactions of electrically conducting fluids and electromagnetic fields. When fluid such as ionized gases (plasma, mercury and molten iron, electrolytes which are only but a few electrically conducting fluids, moves through a magnetic field, consequently a current is induced, and in turn the current interacts with the magnetic field to produce a body force on the fluid. Such forces (current) generated has been used in the generation of electricity by the help of (MHD) designed generators. The analysis of suction effect on fluids through magneto hydro dynamics has attracted many researches due to its application in geothermal and oil reservoir , where it deals with the behaviour of fluids under rest and motion, the phenomenon of unsteady Magneto-hydrodynamics (MHD) flow with suction effect have become very popular and a subject of growing interest due to its application in many engineering and geophysical processes such as cooling of nuclear reactor, geothermal reservoirs, underground energy transport, MHD pump, MHD power generators, and so on. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself. This has many applications in the field of engineering and science and as such attracted considerable attention of engineers and scientists all over the world.

Heat transfer in magnetohydrodynamic (MHD) flows emerge in various innovative applications. The MHD flow in the planar channels prompts a start-up procedure suggesting in this manner a viscous layer at the limit is all of a sudden set into movement and ends up imperative in the utilization of different branches of geophysics, astronomy and liquid building. A field in which MHD will assume a basic part is atomic combination, where it is engaged with no less than two distinct issues: the control and elements of plasma, and the conduct of the fluid metal amalgams utilized in a portion of the right now considered outlines of tritium reproducing covers. A hypothetical investigation of the oscillatory flow of a couple stress impacts, in a rotating,channel, has been directed by [11]. [3] discussed the consistent hydromagnetic flow of a couple stress liquid affected by a uniform magnetic field.

A hypothetical investigation of the heat and mass transfer impacts on an unsteady flow of a couple-stress liquid in a horizontal wavy permeable space with travelling

thermal waves employed by [15]. [13] studied the effect of chemical reaction of a couple stress fluid on an inclined assymetric channel. The oscillatory flow in the optically thin thermal radiation limit has been studied by [5]. The peristaltic motion of a couple stress fluid in a porous medium is studied by [10]. [13] studied the effects of chemical reaction in oscillatory flow and mass transfer in an asymmetric wavy channel. Sunspots are caused by the solar magnetic fields, the sun power is also governed by MHD. The effect of MHD oscillatory flow through porous medium with heat source has been investigated by [12]. [7] studied the effects heat and mass transfer in the oscillatory flow of blood. The oscillatory flow have extraordinary pertinence with applications in oil-penetrating, fabricating, preparing of nourishments, oil investigation, and polymer enterprises. The oscillatory channel flow in viscous fluid is extended to a non-Newtonian Jeffrey fluid model discussed by [1]. [9] analyzed the oscillatory flow in an asymmetric channel under the effects of chemical reaction and heat transfer. Couple pressure fluid theory, created by S[14], is one among the polar fluid theory which considers couple stresses notwithstanding the traditional Cauchy stress.
The cardiovascular framework is delicate to changes in the heart, and flow qualities of blood are adjusted to fulfil changing requests of the life form. For numerous reasons, uses of MHD in physiological stream issues are of developing interest. Oscillatory flow of a fluid and heat transfer with porous under the magnetic field is discussed by [2]. The problem of an oscillatory MHD convective flow in a vertical porous channel is discussed by [4]. Tapered channel may fill in as a model for the intrauterine fluid movement in a sagittal cross-segment of the uterus under tumour treatment. [6] discussed the effects of magnetic field in the peristaltic flow in the tapered-asymmetric channel. However, in this work, due to the porosity of the channel, we shall consider the effects of suction on the flow of the fluid.

2. Formulation of the Problem

We consider the viscous incompressible flow of electrically conducting couple stress fluid in an asymmetric tapered wavy channel.
The geometry of the wall as elucidated by [11] is given as (1).

$$H_1 = -d - m^{'}X - a_1 \sin\left[\frac{2\pi}{\lambda}(X) + \phi\right]$$

$$H_2 = d + m^{'} + a_2 \sin\left[\frac{2\pi}{\lambda}(X)\right] \tag{1}$$

In which a_1 and a_2 are the amplitude of the left and right of the walls and $m^{'}(<< 1)$ is the non-uniform parameter of the channel, ϕ varies in range $0 \leq \phi \leq \pi$. When $\phi = 0$ corresponds to the symmetrical channel.

$a_1,\ a_2,\ d\ and\ \phi$must satisfy the condition of the channel

$$a_1^2 + a_2^2 + 2a_1a_2\cos(\phi)\ \le (2d)^2$$

The governing equations are given by Equation of momentum in (2), (3) & (4):

(2) $$\frac{\partial u'}{\partial t'} - v_0\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{dP}{dX} + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\nu}{K}u' - \frac{\sigma B_0^2}{\rho}u' + g\beta_T(T' - T_0') + g\beta_c(C' - C_0')$$

Energy equation

(3) $$\frac{\partial T'}{\partial t'} - v_0\frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p}\frac{\partial^{2T'}}{\partial y'^2} - \frac{4\alpha^2}{\rho C_p}(T' - T_0')$$

Concentration equation:

(4) $$\frac{\partial C'}{\partial t'} - v_0\frac{\partial C'}{\partial y'} = D\frac{\partial^{2C'}}{\partial y'^2} - K_c'(C' - C_0')$$

The boundary conditions are:

(5) $$y' = H_1,\ \ u' = 0,\ T' = T_1',\ \ C' = C_1'$$

(6) $$y' = H_2,\ \ u' = 0,\ T' = T_0',\ \ C' = C_1'$$

The radiative heat flux is given as (7):

(7) $$\frac{\partial q}{\partial y'} = 4\alpha^2(T_1' - T')$$

where $\alpha^2 = \int_0^\infty K_{\lambda w}\frac{\partial e_{b\lambda}}{\partial t}d\lambda,\ K_{\lambda w}$ is the absorption coefficient, $e_{b\lambda}$ is a plank's function.

2.1. **Non-Dimensionalization of the Governing Equation.** In order to write (2)-(6) in dimensionless form, we use the following dimensionless parameters

$$x = \frac{x'}{\lambda},\ y' = \frac{y'}{d},\ u = \frac{hu'}{\nu},\ t = \frac{\nu t'}{d^2},\ P = \frac{h^2P}{\rho\nu^2},$$

$$Gr = \frac{g\beta_T(T' - T_0')d^3}{\nu^2},\ Gc = \frac{g\beta_C(T' - C_0')d^3}{\nu^2},\ Pr = \frac{\rho C_p\nu}{k},\ \theta = \frac{T' - T_0'}{T_1' - T_0'},$$

(8)

$$\phi = \frac{C' - C_0'}{C_1' - C_0'},\ \delta = \frac{4\alpha^{2h^2}}{\rho C_p\nu},\ \gamma = \frac{\sqrt{K}}{\alpha_{sh}},\ Ha^2 = \frac{\sigma_e B_0^2h^2}{\rho\nu},\ Da = \frac{K}{h^2},\ s = \frac{v_0h}{\nu},\ Sc = \frac{v}{D}$$

The channel wall equations becomes:

$$h_1 = -1 - mx - a\sin\left[2\pi x + \phi\right]$$

(9) $$h_2 = 1 + mx + a_2\sin\left[2\pi x\right]$$

The governing equations in dimensionless form together with appropriate boundary condition are (10), (11) & (12) as follows:

$$\frac{\partial u}{\partial t} - s\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\partial^2 u}{\partial y^2} - \eta\frac{\partial^4 u}{\partial y^4} - \left(M + \frac{1}{Da}\right)u + Gr\theta + Gc\phi \tag{10}$$

$$\frac{\partial \theta}{\partial t} - s\frac{\partial \theta}{\partial y} = \frac{1}{Pe}\frac{\partial^2 \theta}{\partial y^2} + N^2\theta \tag{11}$$

$$\frac{\partial \phi}{\partial t} - s\frac{\partial \phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - K_c\phi \tag{12}$$

The boundary conditions now become:

$$y = h_1,\ u = 0,\ \theta = 1,\ \phi = 1 \tag{13}$$

$$y = h_2,\ u = 0,\ \theta = 0,\ \phi = 0 \tag{14}$$

Stress free conditions $\frac{\partial^2 u}{\partial y^2} = 0$ for $y = h_1,\ h_2$

2.2. **Method of Solution/Solution of the Problem.** As shown by [11], the system of equations can be reduced to closed form (purely oscillation) by assuming:

$$-\frac{dP}{dx} = \lambda e^{i\omega t},\ u\left(y,t\right) = u_0\left(y\right)e^{i\omega t},\ \theta\left(y,t\right) = \theta_0\left(y\right)e^{i\omega t},\ \phi\left(y,t\right) = \phi_0\left(y\right)e^{i\omega t} \tag{15}$$

Now substituting equations (15) into equations (10)-(14), we obtain the following equations:

$$\eta\frac{\partial^4 u}{\partial y^4} - \frac{\partial^2 u_0}{\partial y^2} - s\frac{\partial u_0}{\partial y} - L_2 u_0\left(y\right) = \lambda + Gr\theta_0\left(y\right) + Gc\phi_0(y) \tag{16}$$

$$\frac{\partial^2 \theta}{\partial y^2} + L_3\frac{\partial \theta}{\partial y} + L_4\theta_0\left(y\right) = 0 \tag{17}$$

$$\frac{\partial^2 \phi_0}{\partial y^2} + s\frac{\partial \phi_0}{\partial y} - L_5\phi_0\left(y\right) = 0 \tag{18}$$

where, $L_1 = \left(M + \frac{1}{Da}\right),\ L_2 = L_1 + i\omega$.
The corresponding boundary conditions in equations (13) and (14) become:

$$y = h_1,\ u_0 = 0,\ \theta_0 = 1,\ \phi_0 = 1 \tag{19}$$

$$y = h_2,\ u_0 = 0,\ \theta_0 = 0,\ \phi_0 = 0 \tag{20}$$

Stress free condition is (21) as follows:

$$y = h_1,\ h_2,\ u_0^{''} = 0 \tag{21}$$

Substituting the above boundary conditions (19)-(20) in the equations (16)-(18), we obtain the following solutions:

(22)
$$u\left(y,t\right)=\left(c_5e^{m_5y}+c_6e^{m_6y}+c_7e^{m_7y}+c_8e^{m_8y}+K_0+K_1e^{m_1y}+K_2e^{m_2y}+K_3e^{m_3y}+K_4e^{m_4y}\right)e^{i\omega t}$$

(23)
$$\theta\left(y,t\right)=\left(c_3e^{m_3y}+c_4e^{m_4y}\right)e^{i\omega t}$$

(24)
$$\phi\left(y,t\right)=\left(c_1e^{m_1y}+c_2e^{m_2y}\right)e^{i\omega t}$$

where, $L_1=M+\frac{1}{Da}$, $L_2=L_1+i\omega$, $A_3=sPe$, $L_4=Pe\left(N-i\omega\right)$, $L_5=K_r+i\omega$, $c_1=\frac{e^{m_2h_2}}{e^{m_1h_1+m_2h_2}-e^{m_1h_2+m_2h_1}}$, $c_2=\frac{-c_1e^{m_1h_2}}{e^{m_2h_2}}$, $c_3=\frac{e^{m_4h_2}}{e^{m_3h_1+m_4h_2}-e^{m_3h_2+m_4h_1}}$, $c_4=-\frac{-c_1e^{m_1h_2}}{e^{m_2h_2}}$, $c_5=\frac{A_8-A_{10}c_6}{A9}$, $c_6=\frac{A_{10}e^{m_5}-A_9e^{m_6}}{A_{10}e^{m_5}-A_9e^{m_6}}$, $m_1=\frac{-L_3+\sqrt{L_3^2+4L_4}}{2}$, $m_2=\frac{-L_3-\sqrt{L_3^2+4L_4}}{2}$, $m_3=\frac{-s+\sqrt{s-4L_5}}{2}$, $m_5=L_2$, $m_6=-L_2$, $m_7=\frac{L_2}{\eta}$, $m_8=\frac{-L_2}{\eta}K_0=\frac{\lambda}{L_2}$, $K_1=\frac{-Gcc_1}{\eta m_1^4-m_1^2+sm_1-L_2}$, $K_2=\frac{-Gcc_1}{\eta m_2^4-m_2^2+sm_2-L_2}$, $K_3=\frac{-Grc_3}{\eta m_3^4-m_3^2+sm_3-L_2}$, $K_4=\frac{-Grc_4}{\eta m_4^4-m_4^2+sm_4-L_2}$,

3. Data Presentation and Discussion of Results

Data presentation and discussion of our results are as follows:

3.1. Data Presentation. Data presentation are as expressed in Figures 1 - 6 below:

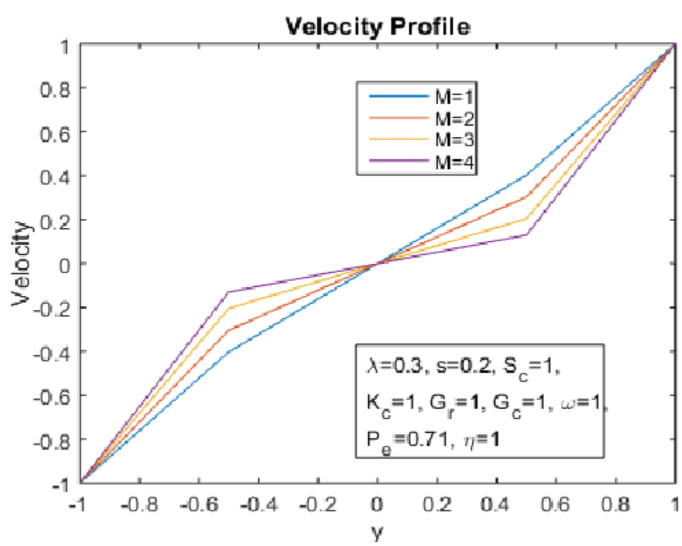

Figure 1: Velocity profile for different values of M

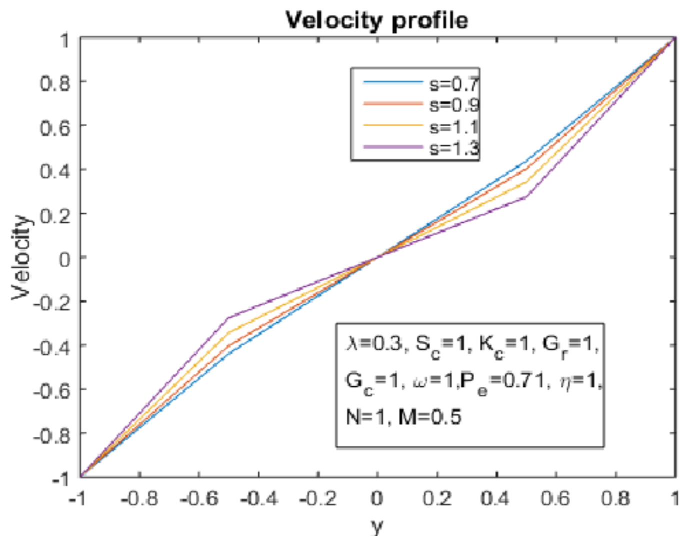

Figure 2: Velocity profile for different values of s

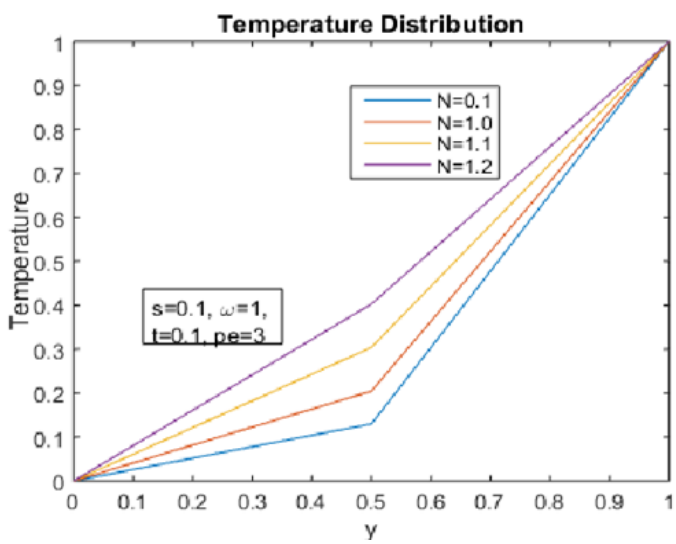

Figure 3: Temperature profile for various values of N

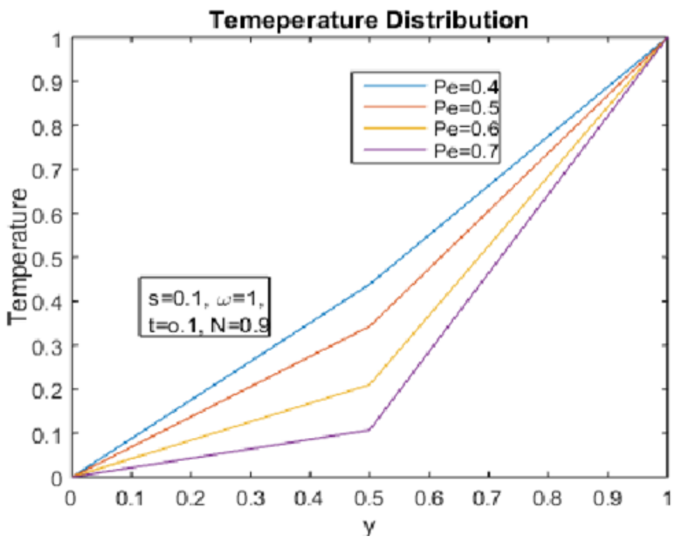

Figure 4: Temperature profile for various values of Pe

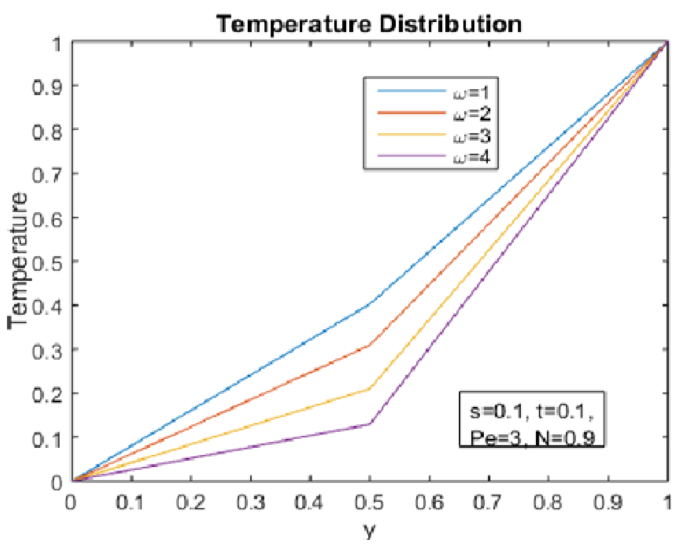

Figure 5: Temperature profile for various values of ω

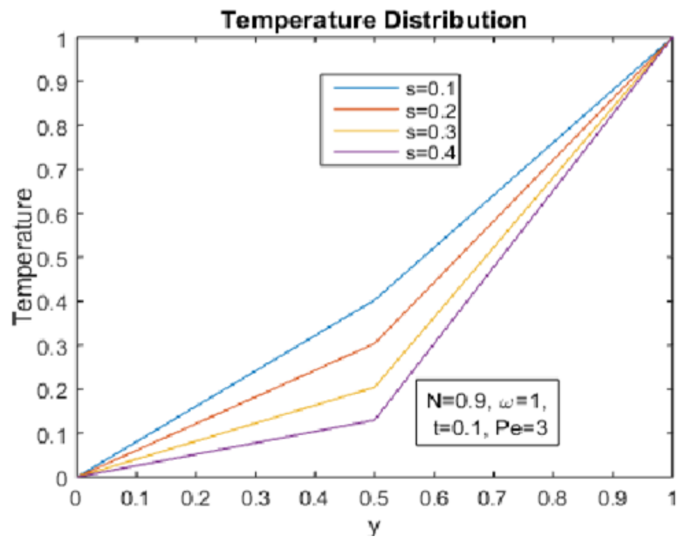

Figure 6: Temperature profile for various values s

3.2. **Discussion of Results.** The magnitude of the fluid velocity varies with different values of Hartmann number M and suction parameter s. These are analysed in Figures 2 and 3. Figure 2 illustrates the drag force effect on fluid flow. The velocity profile decreases with the increment of Hartmann number $(1 \leq M \geq 4)$. The role of Hartmann number which is the magnetic parameter is to suppress turbulence. Physically, when magnetic field is applied to any fluid, the apparent viscosity of the fluid increases to the point of becoming viscous elastic solid. It is of great interest that yield stress of the fluid can be controlled very accurately through variation of the magnetic field intensity. The result is that the ability of the fluid to transmit force can be controlled with help of electromagnet which give rise to many possible control-based applications, including MHD power generation, electromagnetic casting of metals, MHD propulsion and so on. The impact of suction parameter s profile are depicted in Figure. 2. It is clearly seen that velocity profile diminish with the increase $(0.7 \leq S \geq 1.3)$ of S. This is due to the porosity of plates. The magnitude of the fluid temperature is also affected with different values of thermal radiation parameter N, suctionparameter s, Peclet number $Pe$and frequency of oscillation ω. Figure 4 depict temperature fields for increment of thermal radiation parameter$\delta\,(0.1 \leq N \geq 1.2)$. Thermal radiation is known as electromagnetic radiation or the conversion of thermal energy which generates the thermal motion of particles in matter. Thermal radiation could be attributed due to thermal excitation. The temperature field is affected significantly with increase in thermal radiation parameter (N). Thermal radiation for a medium which contains it inevitably has pressure and density gradients, and the treatment requires the use of hydrodynamics. Increase in Peclet number N decreases the temperature distribution. This is clearly shown in Figure 5. Figures 6 and 7 depict the effect of frequency of oscillation ω and suction parameter s on temperature profile respectively. Increase in both the flow parameters decrease the temperature distribution.

Conclusion: Heat and Mass Transfer Effect on MHD Oscillatory Flow of a Couple Stress Fluid in Asymmetric Tapered Channel. The temperatures prescribed at the channel walls are non-uniform. Magnetic field strength which is uniform is applied transversely to the channel. Dimensionless parameters are used to non-dimensionalized the governing equations to dimensionless form. Closed form solution method is used to solve the dimensionless equations govern the flow and the solutions for velocity, temperature and concentration distribution are obtained. The effects of flow parameters on velocity profile and temperature distribution are presented, discussed and shown graphically in details. The results of this work is in agreement with the results obtained in [11] when $s = 0$.
It could be concluded that:

(1) Increase in s accelerates the velocity of the fluid.
(2) Increase in s declines the temperature distribution
(3) Higher values of M retards fluid velocity
(4) Increase in Peclet number and frequency of Oscillations decrease the temperature distribution.

This work can be extended to further studies such as viscous dissipation. This can be done by adding $\left(\frac{\partial u}{\partial y}\right)^2$ to the energy equation.

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