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Analysis of High-Pressure Transient flow in Pipeline of Hydrogen-Natural Gas Mixture

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Abstract

Hydrogen is a high pressure gas whose natural mixture requires accurate prediction of pressure drop. The reduced order modelling is used in analysis of transient flow where viscosity change is neglected leading to Euler equation which becomes the governing equation. This study provides an improvement on the accurate prediction of pressure drop analysis of transient flow of hydrogen natural gas mixture or hydrogen in pipeline. The construction of efficient reduce order model was achieved using implicit Steger-Warming flux vector splitting method (FSM) in which the accuracy and computational efficiency are tested. The result is efficient when compared to normal conventional numerical techniques. This model can, significantly, assist in analysing high-pressure transient flow behaviour at any point during the flow.

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1. INTRODUCTION

Hydrogen is the lightest and most abundant fuel gas in the universe. Hydrogen is globally receiving attention as alternative energy source, been the most cleanness fuel gas hence seen as an important gas to sustain energy generation [1]. Many researches have been conducted on its production, transportation and storage. Hydrogen is known to be mostly transported through existing natural gas pipelines to reduce transportation cost. Hydrogen is known to have low volumetric energy content and hence deserve a high pressure for its transportation [2].

Hydrogen has a societal benefit including the reduction of greenhouse gas effect at zero level [3]. World energy generation was dominated by coal in the 19th century, petroleum oil, natural gas and nuclear energy in the 20th and 21st centuries with a clear indication possibility of hydrogen taking over before the end of 21st century by the recent development [4]. With this hydrogen is contributing positively to world economy mostly on transportation sector going by amount of increase of hydrogen gas car on roads but shall still takes decades before a significant fraction is made compared to cars on roads[5]. Although base on optimization model the hydrogen cars could also have negative impact in the economic growth.

In a step toward realizing hydrogen economy there is need to transport hydrogen from its point of produce to where it can be consumed. To construct a separate pipeline for hydrogen transportation will be more costly for industries therefore there is need of using exiting natural gas pipeline. The transportation cost of hydrogen depends on distance and quantity[6],[7]. The reduction of pressure loss in smooth walls of the pipeline will cut operational cost during transportation up to 10% hence the skin fraction and pressure drag for configuration with reliable computational fluid technique is required in calculation of turbulent flow [8].

The Reduce Order Modelling (ROM) technique can be use in analysis of computational fluid dynamic CFD. This can be achieved by transforming large set of primitive variables into smaller decoupled equations [9]. For accuracy and efficient analysis of transient flow in pipeline ROM technique has demonstrated demonstrated high significant impact [10]. The advantage of ROM is the construction of Eigen analysis in unsteady flows where only few of the original nodes are retained. Computational efficiency is highly demanding in CFD analysis which results to high computational cost. ROM is often use in the analysis of aerodynamic problem and design of wing fuselage system and exhibited a low cost in computation [11]. ROM has been a well-known numerical technique but not much used in the transient flow analysis. In the research by [10] where ROM technique is used remarkable accuracy was achieved in natural gas transient flow analysis with low computational cost. ROM can also be applied in subsonic unsteady flow. The objective of this paper is to improve on accurate prediction of pressure drop analysis of transient flow of hydrogen natural gas mixture or hydrogen in pipeline.

2. METHOD AND MATERIALS

The transient flow is assumed to be an isothermal condition and follows polytropic process governed by Euler equation within the pipeline [12]. The effect of potential energy is ignored with pipeline shear stress and since hydrogen is a high-pressure fluid the flow shall assumed to be compressible with rapid closure of valves in up and down streams of pipeline to meet consumers demand. A one dimensional and homogeneous mixture of hydrogen-natural gas is also assumed thereby the hydrogen fluid mass ratio is used in the resulting mixture. [2], [1].

Governing Equation: From our assumptions mass conservation and momentum equation, Euler fluid equations of motion are given as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + c^2 \rho)}{\partial x} + \frac{f \rho u |u|}{2d} = 0 \tag{2}$$

whereρ and u are fluid mixture density and velocity respectively. C is celerity of pressure wave, with f as pipe friction factor [10].

Mixture equations: The flow is compressible with rapid closure of valves at the upstream and downstream. The analogy of calculating transient pressure drop will be achieved by fixed flow condition with hydrogen fluid mixture mass ratio.

$$\phi = \left[\frac{M_h}{M_h + M_g} \right] \tag{3}$$

where M_h and M_g are mass of hydrogen and natural gas respectively.
By polytropic process, density of the fluid will evolve in the mixture [2].

$$\rho_h = \rho_{h_0} \left(\frac{p}{p_0} \right)^{\frac{1}{n'}} , \ \rho_g = \rho_{g0} \left(\frac{p}{p_0} \right)^{\frac{1}{n''}} \tag{4}$$

are densities of hydrogen and natural gas in the mixture, where ρ_{h_0} , ρ_{g_0} , are densities of hydrogen and natural gas, p, p_0 are transient pressure and permanent pressure respectively [2]. The density of gas mixture shall be expressed has [2].

$$\rho = \left[\frac{\phi}{p_h} + \frac{1-\phi}{p_g} \right]^{-1} = \left[\frac{\phi}{p_{h_0}} \left(\frac{p}{p_0} \right)^{\frac{1}{n'}} + \frac{1-\phi}{p_{g_0}} \left(\frac{p}{p_0} \right)^{\frac{1}{n''}} \right]^{-1} \tag{5}$$

Pressure wave: The mixture fluid is compressible its pressure wave will be defined by [2]

(6)

$$\left[\frac{\phi}{p_{h_0}}\left(\frac{p}{p_0}\right)^{\frac{1}{n'}}+\frac{1-\phi}{p_{g_0}}\left(\frac{p}{p_0}\right)^{\frac{1}{n''}}\right]^{-1}\times\left[\frac{1}{p}\left[\frac{\phi}{n'p_{h_0}}\left(\frac{p}{p_0}\right)^{\frac{1}{n'}}+\frac{1-\phi}{n''p_{g_0}}\left(\frac{p}{p_0}\right)^{\frac{1}{n''}}\right]\right]^{-\frac{1}{2}}$$

2.1. **Formulation of Numerical schemes.** Equations (1) and (2) can be rewritten in matrix form

$$\frac{\partial Q}{\partial t}+\frac{\partial E(Q)}{\partial x}-H(Q)=0 \tag{7}$$

$$Q=\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}=\begin{bmatrix} \rho \\ \rho u \end{bmatrix}$$

$$E(Q)=\begin{bmatrix} q_2 \\ \frac{q_2^2}{q_1}+c^2 q_1 \end{bmatrix}, H(Q)=\begin{bmatrix} 0 \\ -\frac{q_2 f|q_2|}{2q_1 D} \end{bmatrix}$$

$$\frac{\partial E(Q)}{\partial Q}=\begin{pmatrix} 0 & 1 \\ -\frac{q_2^2}{q_1}+c^2 & \frac{2q_2}{q_1} \end{pmatrix}, \frac{\partial H(Q)}{\partial Q}=\begin{pmatrix} 0 & 0 \\ \frac{f q_2|q_2|}{2q_1^2 D} & -\frac{f|q_2|}{q_1 D} \end{pmatrix}, A=\frac{\partial E}{\partial Q}, B=\frac{\partial H}{\partial Q}.$$

$$A^+=\begin{pmatrix} \frac{c^2-u^2}{2c} & \frac{c+u}{2c} \\ \frac{c(c+\frac{p}{c_2}\cdot\frac{\partial c}{\partial p})}{2c} & \frac{(u+c)^2}{2} \end{pmatrix}, A^-=\begin{pmatrix} \frac{u^2-c^2}{2c} & \frac{c-u}{2c} \\ \frac{-c(-c-\frac{p}{c_2}\cdot\frac{\partial c}{\partial p})}{2} & \frac{(c-u)^2}{2} \end{pmatrix}, B=\begin{pmatrix} 0 & 0 \\ 0 & \frac{uf}{2d} \end{pmatrix}.$$

Rewriting (7) and taking time difference approximation:

$$Q_t=-(E_x+H)$$

$$Q_{n+1}=Q_n+\frac{\Phi}{1-\varepsilon}\Delta t\left[\left(\frac{\partial Q}{\partial t}\right)^{n+1}+\left(\frac{\partial Q}{\partial t}\right)^{n}\right].$$

Substituting we have:

$$Q_{n+1}=\Delta t\left[\left(\frac{\partial E}{\partial x}+H\right)^{n+1}+\left(\frac{\partial E}{\partial x}+H\right)^{n}\right]. \tag{8}$$

Linearizing using Taylor series expansion () term by term:

$$E^{n+1}=E^n+\Delta t\left(\frac{\partial E}{\partial x}\right)^n+O(\Delta t)^2$$

$$E^{n+1}=E^n+A^n(Q_{n+1}-Q_n). \tag{9}$$

Substituting Using implicit Steger-warming flux vector splitting method (FSM) as numerical scheme and backward time Euler difference finite deformation [13].

$$-\left(\tfrac{\Delta t}{\Delta x}A^{+}_{j-1}\right)\Delta Q_{j-1}+\left(1+\tfrac{\Delta t}{\Delta x}(A^{+}_{j}-A^{-}_{j})-\Delta t B_j\right)\Delta Q_j+\left(\tfrac{\Delta t}{\Delta x}A^{-}_{j+1}\right)\Delta Q_{j+1}$$
$$=-\tfrac{\Delta t}{\Delta x}(E^{+}_{j}-E^{+}_{j-1}+E^{-}_{j+1}-E^{-}_{j+1})+\Delta t H_j$$

Linearism of equation (8) so as to enable the eigen analysis and reduce order modelling of the gas mixture at every time step by considering

$$Q^{n+1}=Q^{0}+\hat{Q}^{n+1}, \tag{10}$$

in which $Q^0, \hat{Q}^{n+1}$ are steady and small approximated results. Substituting and evaluating into (3) we get:

$$-\left(\frac{\Delta t}{\Delta x}A^{0}_{j+1}\right)\hat{Q}^{n+1}_{j-1}+\left(1+\frac{\Delta t}{\Delta x}(A^{0+}_{j}-A^{0-}_{j})-\Delta t B^{0}_{j}\right)\hat{Q}^{n+1}+\left(\frac{\Delta t}{\Delta x}A^{0-}_{j+1}\right)Q^{n+1}_{j+1}=\hat{Q}^{n}_{j}. \tag{11}$$

Let $\hat{Q}=x_j\exp(\lambda_i t),\ z_i=\exp(\lambda_i\Delta)$ where λ_i and x_i are the respective eigenvalue and eigenvectors at time-step. Generalising and substituting in the equation we have:

$$ZW^{n}X=IX; \tag{12}$$

where Z and X are the diagonal matrix of eigenvalue at each time step and corresponding eigenvector. And the left eigenvector

$$(W^{0})^{T}YZ=IY \tag{13}$$

The behaviour fluid flow will be

$$\hat{Q}=X\hat{c} \tag{14}$$

where $\hat{c}$ is the vector of the normal mode coordinate [1]
Substituting we get:

$$\hat{c}^{n+1}=Z\hat{c}^{n}+Y^{T}V^{n+1}. \tag{15}$$

Equation (15) is the ROM of equation (7).

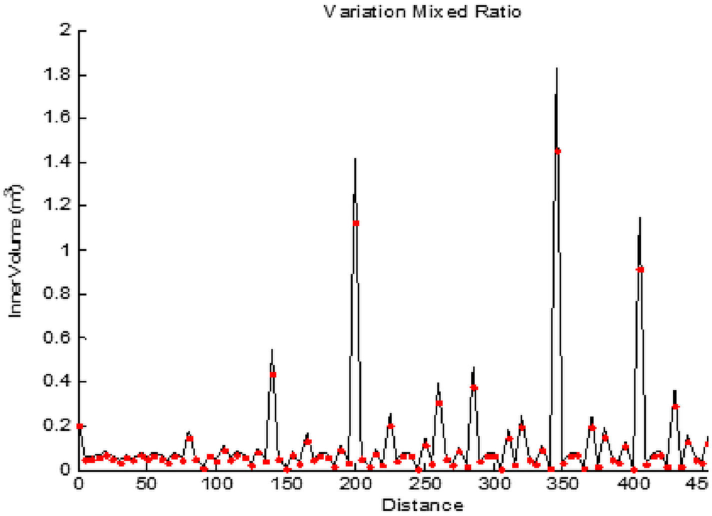

Figure 1: Gas inner volume per distance

The velocity profile shows increase for increase on hydrogen presence. The peak point is find when the pipeline distance and with velocity change at $0.45m/s$ at 20 percentage presence of hydrogen, while that of 25 percentage hydrogen is at the same distance but the peak is $0.35m/s$.

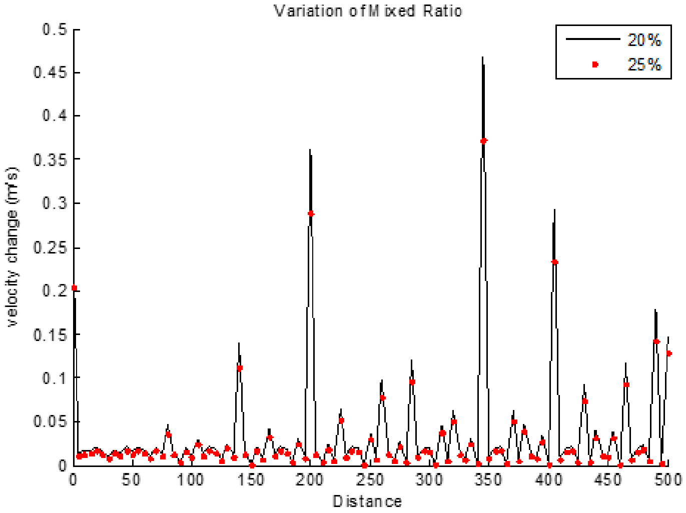

Figure 2: Velocity change with respect to pipeline distance

The volume of gas in pipeline during simulation is analysed. The gas mixture volume continue to increases at get to the peak of about $1.9m^3$ when its flow at distance of 348 meters of the pipe. From Figure 1 the volume state decreasing immediately after peak is achieved this will help in the gas delivery to end users.

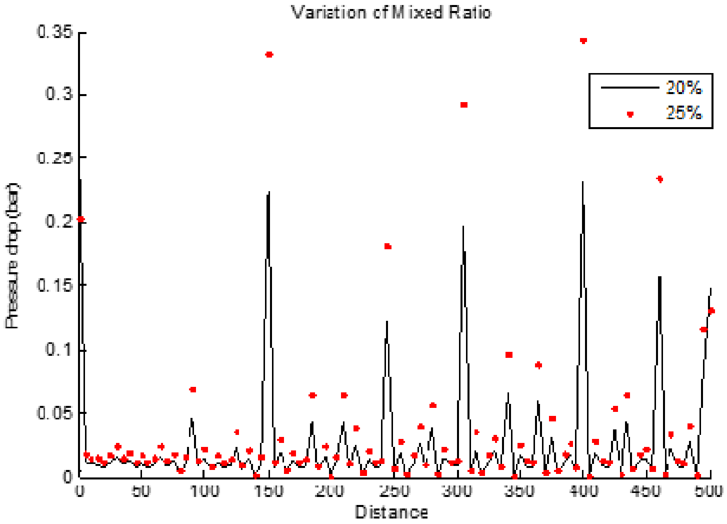

Figure 3: Pressure drop at every point on the pipeline

Pressure is important in the analysis of fluid flow, the effect at point of the pipeline is present. There is significant change on pressure drop for increase on hydrogen present this could be as a result of change on gas mixture density. There are three at which the peak of pressure drop is rich through the pipeline.

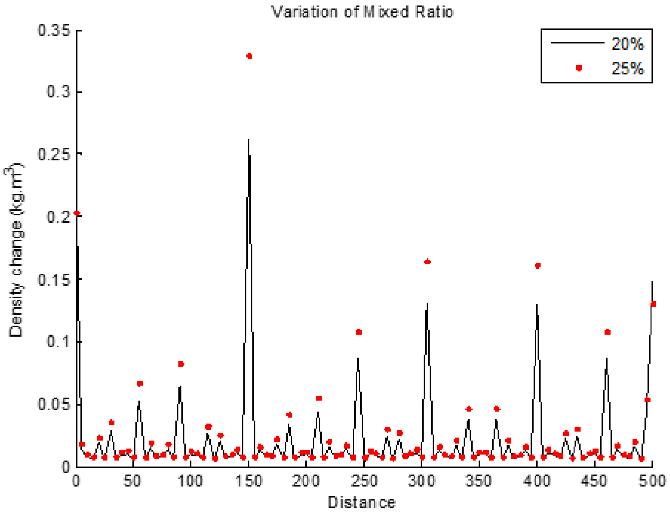

Figure 4: Density of gas mixture

Density is significant in flow behaviour and connected to velocity profile. From Figure 4 peak of mixture density is observed at earlier distance than the other fluid flow parameters, which is at 150 meters distance of 0.25 for 20% hydrogen and 0.34 for 25% hydrogen. The density change increases for increase on hydrogen presence in the mixture.

The celerity wave of a transient is a parameter governed by pressure behaviour which can be observed from Figure 5 and Figure 3. The peak when hydrogen ratio is 25% and 20% are as follow at distances of 150 and 400 meters at 2.3 and 1.5 respectively.

Results and Discussions. In the analysis an iron pipeline is used with dimension 0.4m and 500m diameter and length respectively. The mixture ratio is varying to test its effect to other parameters, when equation (2.4) reduces hence therefore becomes [1]. In the analysis using the same condition, shows the similarity in the present with [1]. We notice pressure drop evolution influence for different mixed ratio has also shown in [2], in Figure 3 with peak drop of .24 bar in the case of and 2.67 respectively. The result shows the same pattern of transient analysis for different mixture ratio. In Figure 3 the variation in density show a similarity in graph with the pressure drop analysis. Since hydrogen is low density gas, mixture density first drop before it beginning to normalized in the transient. This shows a significant effect of density in the transient mixture. From the graphs it can be observed that increase in natural gas present decreases the

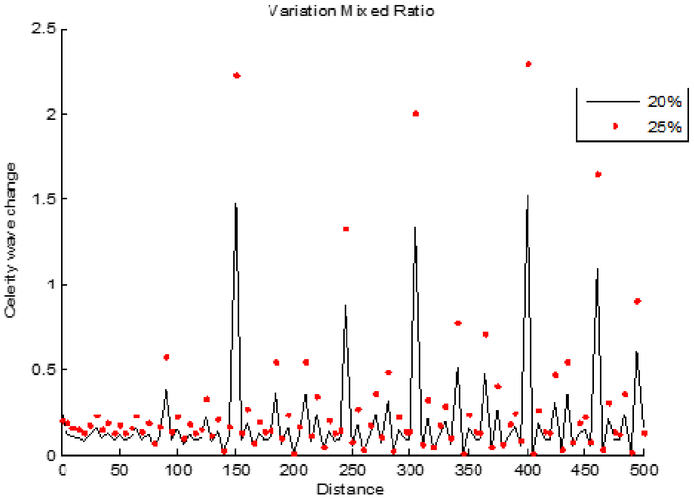

Figure 5: Celerity wave of the transient flow

velocity drop, inner volume while in pressure drop, density change and celerity wave effect increases as natural gas increases. For future studies we shall always look at the density behaviour on transient analysis. □.

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