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A Mathematical Model for Measles incorporating Public enlightenment, Vaccination with Testing and Measles Therapy

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ABSTRACT

A deterministic mathematical model of measles infection incorporating public enlightenment, vaccination of susceptible humans, with testing and measles therapy of exposed humans, with testing and measles therapy of exposed human is developed. The disease-free equilibrium (DFE) state of the model was determined and used to compute the basic reproduction numberR_0, as a threshold for effective disease management. The result from stability analysis for the disease-free equilibrium state (DFEs) shows that it is locally as well as globally asymptotically stable whenever the basic reproduction number R_0, is less than unity $(R_0 < 1)$. Further, the effect of the model parameters on the dynamics of the disease is analyzed using the normalized forward sensitivity indices. Numerical simulations show that, expanded and improved testing and measles therapy is crucial in decreasing measles burden. Furthermore, from the numerical simulations and results it is recommended that a combination of public enlightenment campaigns, vaccination with testing and measles therapy are vital to public health strategies in eradicating measles infection and deaths in the shortest possible time.

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1. Introduction

Measles is a worldwide concern, extremely contagious but vaccine-preventable disease caused by a virus in the paramyxovirus family and it is normally spread through direct contact with infected nasal and throat secretion and through the air (coughing and sneezing) [9, 19, 26]. Many countries around the world are experiencing measles outbreaks. Outbreaks are ongoing in Madagascar and Nigeria among other African countries as reported by World Health Organization [4, 24] As at $5th$ November, 2019, there was a total of $413,308$ confirmed cases reported to World Health Organization (WHO) by 187 members state [24]. The disease has continued causing both economic and health problem [11].

Several measles-connected mortality are caused by complications associated with the disease. Serious complication are more among children under the age of five years or adults over the age of thirty. The most serious complications include blindness, encephalitis (an infection that causes brain swelling), severe diarrhea and related dehydration, ear infections, or severe respiratory infections such as pneumonia. A mortality of about $142,000$ people from measles infection was reported in 2018 despite the availability of safe and effective vaccine [24, 14].

According to [22] poor vaccination coverage and large pockets of unvaccinated young children due to conflicts, security or breakdown in services making it hard to reach children in remote or hard-to-reach areas have resulted in devastating measles outbreaks in many parts of the world. [7], stated that not every administration of a vaccine confer immunity, and protection from vaccine can wane over time thus, making the infectious disease endemic in many part of the world. Any non-immune person (who has not been vaccinated or was vaccinated but did not develop immunity) can become infected. Much more, parents are not vaccinating their children due to complacency, mistrust and misinformation about vaccines poses great impediment to measles eradication[22].

Based on current trends of measles incidence and resurgent in a number of countries, elimination is under threat. This clearly calls for a coordinated control strategies to help combat this threat – the thrust of this paper.

A number of mathematical modeling studies have been carried out in recent time to quantify measles infection burden with emphasis on vaccination [1, 2, 8, 17, 18, 20, 21]. Modelling measles infection with testing and measles therapy as intervention strategy can be seen in [15, 16], but little or no study has been done to investigate or quantify measles infection burden incorporating public enlightenment campaigns, vaccination with testing and measles therapy. Thus, it is informative to carry out modeling studies incorporating the three intervention strategies.

This paper is organized as follows: The model is formulated in section 2. The local and global analysis of the model is explored in section 3. Sensitivity analysis of the basic reproduction number is carried out in section 4. Numerical simulations and results is presented in section 5. Discussion of results is carried out in section 6 while concluding remarks are made in section 7.

2. THE BASIC MODEL FORMULATION

The total population at timet, denoted by N(t) is subdivided into five classes namely: susceptible humans $S(t)$, exposed (latent) $E(t)$, infectious $I(t)$, vaccinated $V(t)$, and Recovered humans $R(t)$. So that;

$$N(t) = S(t) + E(t) + I(t) + V(t) + R(t)$$

The variables and parameters used in the model are defined in Table 1 and Table 2.

TABLE 1. Description of model variables.

Variables	Description
$S(t)$	The number of susceptible humans at time t
$E(t)$	The number of exposed (latent) humans at time t
$I(t)$	The number of infectious humans at time t
$V(t)$	The number of vaccinated humans at time t
$R(t)$	The number of recovered humans at time t

TABLE 2. Description of model parameters.

Parameter	Description
θ	Birth rate
ω	Infant born without passive immunity
α	Infection transmission rate
ρ	The efficacy of public enlightenment
ε	Progression rate from latent to infectious class
μ	Natural mortality rate
ψ	Measles testing and therapy for exposed class
τ	Recovery rate
κ	Measles - induced mortality rate
δ	Waning immunity rate
γ	vaccination rate

From the above definition of variables (Table 1) and parameters $(Table2)$, the interactions and flow in the different compartments are as depicted in the schematic flow diagram $(Figure1)$.

The susceptible population $S(t)$, is generated by birth of individuals into the population at the rate $\theta\omega$, (where ω is the rate of infants born without passive immunity) and waning

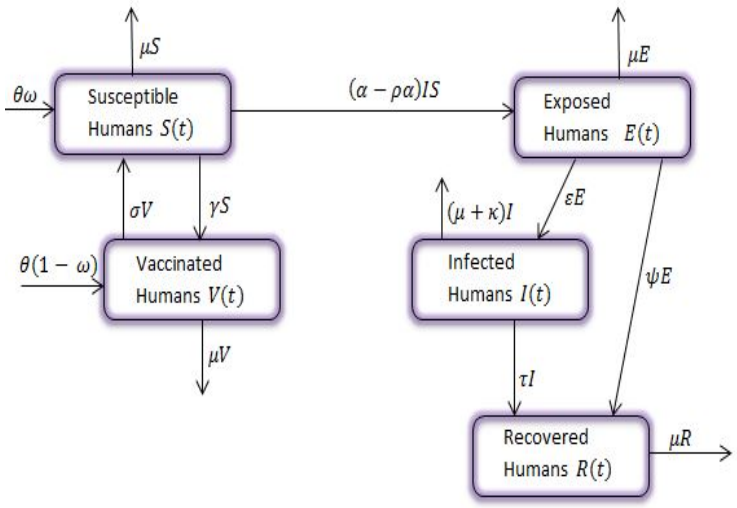


Figure 1: Schematic description of the mathematical model

of immunity (passive immunity and poor vaccine) of vaccinated individuals. It reduced by infection, the rate α. The rate $\rho\alpha$ $(0 < \rho < 1)$ is the reduced rate of measles infection due to public enlightenment, the parameterρ measure the efficiency of public enlightenment (mass immunization campaigns). The susceptible population is further reduced by vaccination at the rate γ and by natural death at the rate μ. Putting all these assumptions and definition together gives the rate of change of the susceptible population as

$$\frac{ds}{dt} = \theta\omega + \sigma V - (\alpha - \rho\alpha)IS - (\gamma + \mu)S.$$

The population of exposed (latent) humans $E(t)$ is generated following infection (at the rates $(1 - \rho)\alpha$. They are decreased as a result of progression into the infectious class at the rate ϵ, testing and measles therapy at the rate ω and natural death at the rate μ, so that

$$\frac{dE}{dt} = (\alpha - \rho\alpha)IS - (\varepsilon + \psi + \mu)E.$$

Infectious humans$I$ are generated as a result of progression into the infected class from the exposed class at the rate ε. It is diminished by recovery at the rate τ, measles-induced death at the rate κ and natural death at the rate μ. Hence,

$$\frac{dI}{dt} = \varepsilon E - (\tau + \kappa + \mu)I.$$

The population of vaccinated individuals with immunity $V(t)$ is generated by birth of infants with passive immunity (it is assumed that their mothers were vaccinated or have

recovered form measles infections) at the rate $(1-\omega)\theta$ and vaccination of susceptible individuals at the rate γ. They are decreased as a result of waning immunity (passive immunity and incomplete dose of **MMR** vaccine) of vaccinated individuals at the rate σ, and natural death at the rate μ, so that

$$\frac{dV}{dt} = \theta(1-\omega) + \gamma S - (\sigma+\mu)V.$$

The population of recovered individuals $R(t)$ are generated as a result of successful testing and measles therapy of exposed individuals at the rate ψ, and recovery of infectious individuals at the rate τ. They are decreased natural death at the rate μ. Hence,

$$\frac{dR}{dt} = \tau I + \psi e - \mu R.$$

2.1. **The Model Equation.** .

$$\frac{ds}{dt} = \theta\omega + \sigma V - (\alpha - \rho\alpha)IS - (\gamma+\mu)S, \tag{1}$$

$$\frac{dE}{dt} = (\alpha - \rho\alpha)IS - (\varepsilon + \psi + \mu)E \tag{2}$$

$$, \frac{dI}{dt} = \varepsilon E - (\tau + \kappa + \mu)I, \tag{3}$$

$$\frac{dV}{dt} = \theta(1-\omega) + \gamma S - (\sigma+\mu)V. \tag{4}$$

$$\frac{dR}{dt} = \tau I + \psi e - \mu R. \tag{5}$$

2.2. **Invariant Region.** We obtain the invariant region in which the model solution is bounded. All the associated parameters and state variables are non-negatives for $t \geq 0$. Consider the biological feasible region

$$\triangle = \{(S,E,I,V,R)\epsilon\Re^5 : N \leq \frac{\theta}{\mu}\}$$

Lemma 2.1. *The closed set Ω is positively and attracting with respect to the system of equations $(1)-(5)$.*

Proof: Adding equations $(1)-(5)$ gives the rate of change of the total population:

$$\frac{dN}{dt} = \theta - \mu N - \kappa I \tag{6}$$

It is clear from equation(6) that

$$\frac{dN}{dt} = \theta - \mu N$$

it follows that
$\frac{dN}{dt} \leq 0$ if $N(t) \geq \frac{\theta}{\mu}$
Thus, by a standard comparison theorem (Lakshmikantham et al, 1989) can be used to show that

$N(t) \leq N(0)e^{-\mu t} \leq \frac{\theta}{\mu}(1 - e^{-\mu t})$ in particular, $N(t) \leq \frac{\theta}{\mu}$ if $N(0) \leq \frac{\theta}{\mu}$.

Thus, the region Ω ispositively-invariant. However if $N(t) \leq \frac{\theta}{\mu}$, then either the solution enters Ω in finite time, or $N(t)$ approaches $\frac{\theta}{\mu}$ asymptotically. Hence the region $\triangle$ attracts all solutions in $\mathbb{R}^5$.

Therefore, it is sufficient to consider the dynamics of the flow generated by equations $(1) - (5)$ in Ω, where the usual existence, uniqueness, continuation results hold for the system $(1) - (5)$, that is the system is mathematically and epidemiological well-posed in $\triangle$.

3. Analysis of the Model

3.1. **Basic Reproduction Number R_0.** The model $(1) - (5)$ has a disease-free equilibrium **(DFE)** given by

(7)
$$\varepsilon_0 = (S^*, E^*, I^*, V^*, R^*) = \left((\frac{\theta\omega(\delta+\mu) + \delta\theta(1-\omega)}{[(\gamma+\mu)(\delta+\mu) - \delta\gamma]}), 0, 0, \frac{\theta(1-\omega)[(\gamma+\mu)(\delta+\mu) - \delta\gamma] + \gamma[\theta\omega(\delta+\mu) +}{(\delta+\mu)[(\gamma+\mu)(\delta+\mu) - \delta\gamma]} \right.$$

We will study the local stability of the disease free equilibrium state, ε_0, exploring the basic reproduction number R_0. The basic reproduction, R_0, measures the average number of new infection generated by a single infected individual in a completely susceptible population.
For the recipe on computation of basic reproduction number using the next generation operator method, see [3, 6, 10, 22]. Let the non-negative matrix, F, of new infection terms and the M-matrix, V, of transfer terms associated with the model $(1) - (5)$ are given respectively, by

$$F = \begin{pmatrix} 0 & (\alpha - \rho\alpha)S^* \\ 0 & 0 \end{pmatrix}$$

and

$$V = \begin{pmatrix} (\varepsilon+\psi+\mu) & 0 \\ -\varepsilon & \tau+\kappa+\mu \end{pmatrix}$$

Now

$$V^{-1} = \begin{pmatrix} \frac{1}{\varepsilon+\psi+\mu} & 0 \\ \frac{\varepsilon}{(\varepsilon+\psi+\mu)(\tau+\kappa+\mu)} & \frac{1}{\tau+\kappa+\mu} \end{pmatrix}$$

so that

$$FV^{-1} = \begin{pmatrix} \frac{(\alpha-\rho\alpha)S^*}{(\varepsilon+\psi+\mu)(\tau+\kappa+\mu)} & \frac{\alpha-\rho\alpha)S^*}{\tau+\kappa+\mu} \\ 0 & 0 \end{pmatrix}$$

The denominant eigenvalue of $FV^{-1} = \begin{pmatrix} \frac{(\alpha-\rho\alpha)S^*}{(\varepsilon+\psi+\mu)(\tau+\kappa+\mu)} & \frac{\alpha-\rho\alpha)S^*}{\tau+\kappa+\mu} \\ 0 & 0 \end{pmatrix}$ is given by

$$R_0 = \frac{(\alpha-\rho\alpha)S^*}{(\varepsilon+\psi+\mu)(\tau+\kappa+\mu)} \tag{8}$$

or

$$R_0 = \frac{(\alpha-\rho\alpha)[\theta\omega(\delta+\mu)+\delta\theta(1-\omega)]}{(\varepsilon+\psi+\mu)(\tau+\kappa+\mu)[(\gamma+\mu)(\delta+\mu)-\delta\gamma]} \tag{9}$$

3.2. **Local Stability of Disease Free Equilibrium (DFE) State.** We investigate the local stability of the disease free (DFE) state by evaluating the associated Jacobian of equations $(1)-(5)$ at the DFE state. The Jacobian matrix J for the system $(1)-(5)$, evaluated at the disease-free equilibrium, E_0 is given by

$$J(E_0) = \begin{pmatrix} -(\gamma+\mu) & 0 & -(\alpha-\rho\alpha)S^* & \delta & 0 \\ 0 & -(\varepsilon+\psi+\mu) & (\alpha-\rho\alpha)S^* & 0 & 0 \\ 0 & \varepsilon & -(\tau+\kappa+\mu) & 0 & 0 \\ \gamma & 0 & 0 & -(\delta+\mu) & 0 \\ 0 & \psi & \tau & 0 & -\mu \end{pmatrix} \tag{10}$$

For ease of analysis, we perform the following operations.
(1) Subtract column 4 from column 1.
(2) Add row 1 to row 4.
By these operations, we have the following equivalent matrix:

$$J_1(E_0) = \begin{pmatrix} -(\gamma+\mu+\delta) & 0 & -(\alpha-\rho\alpha)S^* & \delta & 0 \\ 0 & -(\varepsilon+\psi+\mu) & (\alpha-\rho\alpha)S^* & 0 & 0 \\ 0 & \varepsilon & -(\tau+\kappa+\mu) & 0 & 0 \\ 0 & 0 & -(\alpha-\rho\alpha)S^* & -\mu & 0 \\ 0 & \psi & \tau & 0 & -\mu \end{pmatrix} \tag{11}$$

Theorem 3.1. *The DFEs of the model (1)-(5), given by E_0, is locally asymptotically stable (LAS) if $R_0 < 1$ and E_0 is unstable if $R_0 > 1$.*

Proof It suffices to show that all the eigenvalues of the characteristic equation of the Jacobian matrix $J_1(E_0)$, have negative real parts. The eigenvalues are given by

$$(-(\gamma+\mu+\delta)-\lambda)(-\mu-\lambda)[(-(\varepsilon+\psi+\mu)-\lambda)(-(\tau+\kappa+\mu)-\psi)-\varepsilon(\alpha-\rho\alpha)S^*] = 0$$

$$(-(\gamma+\mu+\delta)-\lambda)(-\mu-\lambda)[(\varepsilon+\psi+\mu)(\tau+\kappa+\mu)+\lambda(\varepsilon+\psi+\mu)+\lambda(\tau+\kappa+\mu)+\lambda^2-\varepsilon(\alpha-\rho\alpha)S^*] = 0$$

that is

(12)

$$(-(\gamma+\mu+\delta)-\lambda)(-\mu-\lambda)[\lambda^2+\lambda(\varepsilon+\psi+\tau+\kappa+2\mu)+(\varepsilon+\psi+\mu)(\tau+\kappa+\mu)-\varepsilon(\alpha-\rho\alpha)S^*]$$

Then $\lambda = -\mu$ (twice,) $\lambda = -(\gamma+\mu+\delta)$
and

$$[\lambda^2 + \lambda(\varepsilon+\psi+\tau+\kappa+2\mu) + (\varepsilon+\psi+\mu)(\tau+\kappa+\mu) - \varepsilon(\alpha-\rho\alpha)S^*] = 0 \quad (13)$$

Obviously, three eigenvalues are negative. Now equation (13) is the characteristic equation of the sub matrix J_2, where

$$J_2 = \begin{pmatrix} -(\varepsilon+\psi+\mu) & (\alpha-\rho\alpha)S^* \\ -\varepsilon & -(\tau+\kappa+\mu) \end{pmatrix} \quad (14)$$

If the trace of $J_2 < 0$ and the $det(J_1) > 0$ then the eigenvalues are negative The trace of

$$J_2 = -((\varepsilon+\psi+\tau+\kappa+2\mu) < 0 \quad (15)$$

and

$$detJ_2 = -(\varepsilon+\psi+\mu(\tau+\kappa+\mu) - \varepsilon(\alpha-\rho\alpha)S^* > 0 \quad (16)$$

that is

$$1 - \frac{\varepsilon(\alpha-\rho\alpha)S^*}{(\varepsilon+\psi+\mu(\tau+\kappa+\mu} > 0 \quad (17)$$

$1 - R_0 > 0$ if $-R_0 < 1$

3.3. **Global Asymptotical Stability (GAS) of Disease Free Equilibrium (DFE) State.** To ensure that the measles infection eradication is independent of initial sizes of the population of the model, it is imperative to show that the DFE of the model $(1) - (5)$, given by E_0, is globally asymptotically stable.(GAS). This is done now.

Theorem 3.2. *The DFE of model $(1) - (5)$, given by Ω_0 is GAS whenever $R_0 \leq 1$.*

Proof Consider the Lyapunov function

$$P = \varepsilon E + (\varepsilon + \psi + \mu)I$$

with Lyapunov derivative (where a prime represents differentiation with respect to t)

$$\begin{aligned} P' &= \varepsilon[(\alpha - \rho\alpha)IS^* - ((\varepsilon + \psi + \mu)E] + ((\varepsilon + \psi + \mu)[(\tau + \kappa + \mu)I \\ &= \varepsilon(\alpha - \rho\alpha)IS^* - (\varepsilon + \psi + \mu)(\tau + \kappa + \mu)I \\ &= [\varepsilon(\alpha - \rho\alpha)S^* - (\varepsilon + \psi + \mu)(\tau + \kappa + \mu)]I \end{aligned}$$

$$= (\varepsilon + \psi + \mu)(\tau + \kappa + \mu)[(\frac{\varepsilon(\alpha - \rho\alpha)S^*}{(\varepsilon + \psi + \mu)(\tau + \kappa + \mu)} - 1)]I$$

$$= (\varepsilon + \psi + \mu)(\tau + \kappa + \mu)I[R_0 - 1]$$

≤ 0 for $R_0 \leq 1$

It follows from the LaSalle invariance principle [13] that every solution to the equations $(1) - (5)$ with initial conditions inR^5, approaches E_0, as $t \to \infty$, for $R_0 \leq 1$.

4. Sensitivity analysis of R_0 with respect to the control parameters

We carried out sensitivity analysis on the basis of the control parameters, α,ρ,γ and ψ, by the normalized forward sensitivity indices [5, 25] using the following formula:

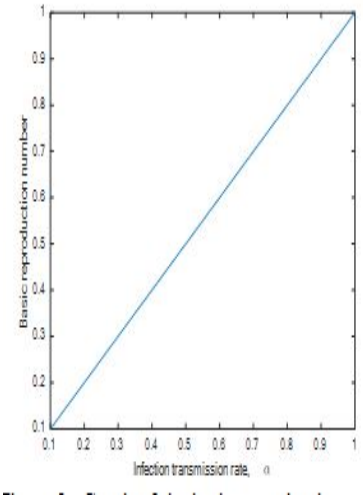
$$\Lambda_\nu^{R_0} = (\frac{\delta R_0}{\delta \nu})(\frac{\nu}{R_0})$$

where ν denotes the model parameter.

The sensitivity index of R_0, with respect to each control parameter is given in Table 3 and the associated graph is shown in Figure 2- Figure 5.

Table 3. Sensitivity indices of R_0.

Parameter	Sensitivity indices
α	1
ρ	-1.5000
γ	-0.8062
ψ	-1.9607



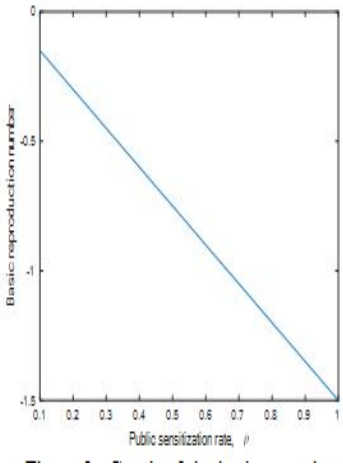
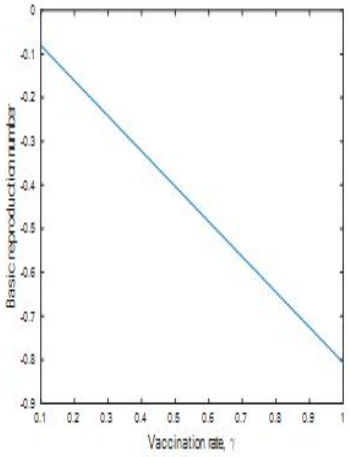


Figure 2: Graph of the basic reproduction R_0, for increasing values of the infection transmission $\alpha \in [0.1, 1]$ and

Figure 3: Graph of the basic reproduction R_0 for increasing values of the efficacy of public enlightenment $\rho \in [0.1, 1]$



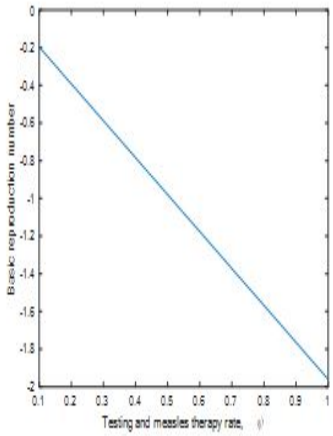


Figure 4: Graph of the basic reproduction R_0 for increasing values of the vaccination rate $\gamma \in [0.1, 1]$ and

Figure 5: Graph of the basic reproduction R_0 for increasing values of the testing and measles therapy, $\psi \in [0.1, 1]$

5. NUMERICAL SIMULATIONS AND RESULTS

Various numerical simulations are carried out to illustrate the theoretical results in this paper. The parameter values for the simulation, unless otherwise stated is given in Table 4.

TABLE 4. Baseline Parameter values for equations $(1) - (5)$

Parameter	Baseline value	Reference
θ	0.02755	Tawhir (2012)
ω	0.5	Assumed
α	0.09091	Fred et al (2014)
ρ	0.6	Assumed
ε	0.125	Tawhir(2012)
μ	0.001253173	Memon et al (2020)
ψ	0.25	Assumed
τ	1/3	WHO (2019)
κ	0.02	Ochoche and Gweryina (2014)
δ	0.167	Stephen et al (2015)
γ	0.7	Assumed

The numerical results are shown in Figure 6 - Figure 12

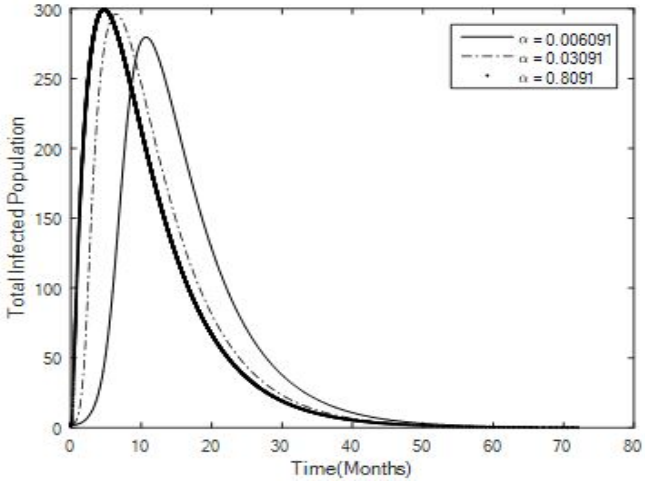


Figure 6: Graph of infected humans as a function of time with no control ($R_0 < 1$) varying infection transmission rate $\alpha_1 = 0.006091; \alpha_2 = 0.03091; \alpha_3 = 0.8091$

All other parameters used are as given in Table 4.

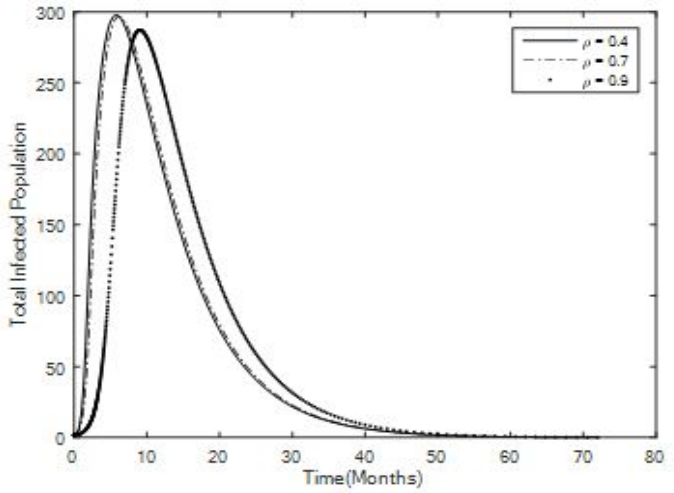


Figure 7: Effect of public enlightenment without other controls.$\rho_1 = 0.04; \rho_2 = 0.7; \rho_3 = 0.9$

All other parameters used are as given in Table 4.

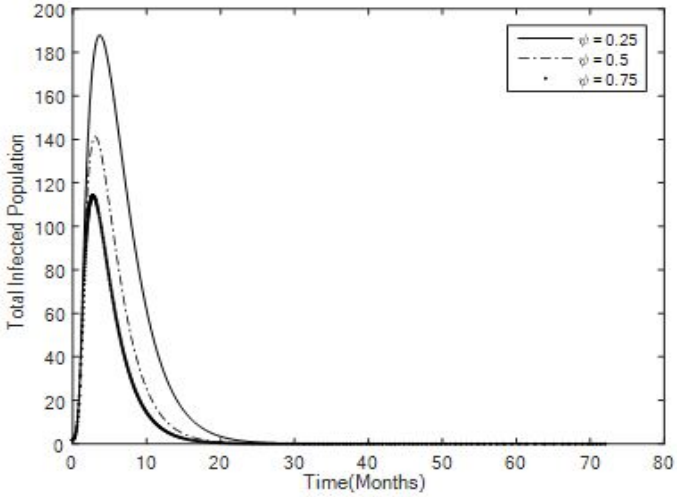


Figure 8: Effect of testing and measles therapy without other controls $\psi_1 = 0.25; \psi_2 = 0.5; \psi_3 = 0.75$

All other parameters used are as given in Table 4.

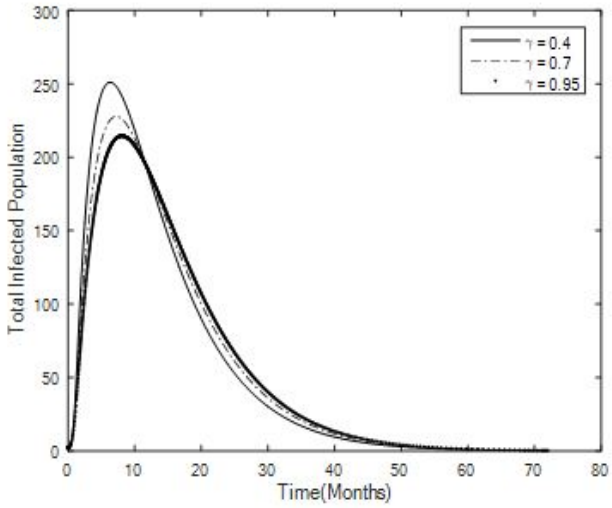


Figure 9: Effect of vaccination of susceptible humans without other controls. $\gamma_1 = 0.4; \gamma_2 = 0.7; \gamma_3 = 0.95$

All other parameters used are as given in Table 4.

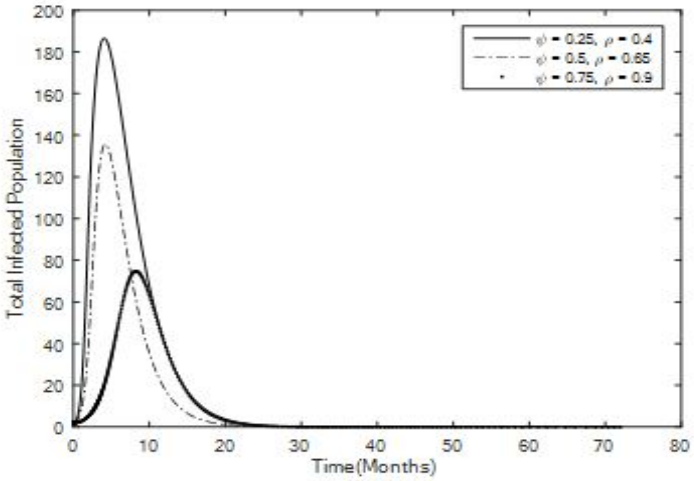


Figure 10: Effect of Testing and measles therapy and public enlightenment without vaccination of susceptible humans $(\psi_1 = 0.25; \rho_1 = 0.4)$; $(\psi_2 = 0.5; \rho_2 = 0.65)$; $(\psi_3 = 0.75; \rho_3 = 0.95)$

All other parameters used are as given in Table 4.

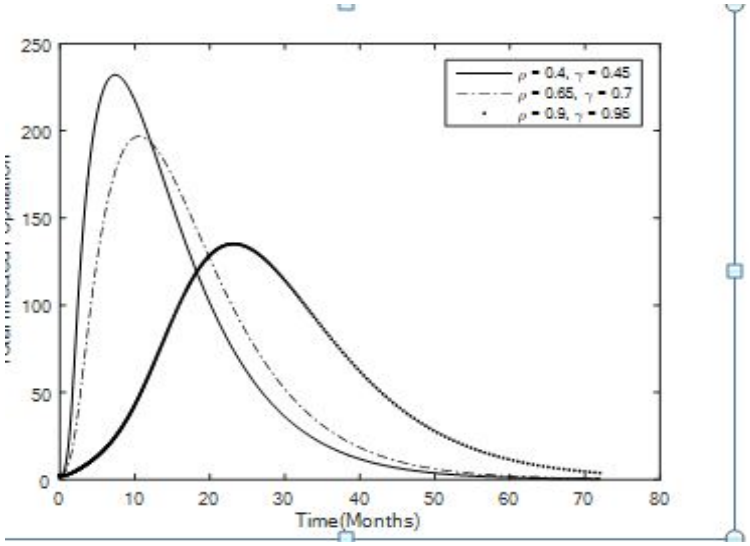


Figure 11: Effect of vaccination of susceptible humans and public enlightenment without testing and measles therapy.
$(\rho_1 = 0.4, \gamma_1 = 0.45); (\rho_2 = 0.65, \gamma_2 = 0.7); (\rho_3 = 0.9, \gamma_3 = 0.95)$

All other parameters used are as given in Table 4.

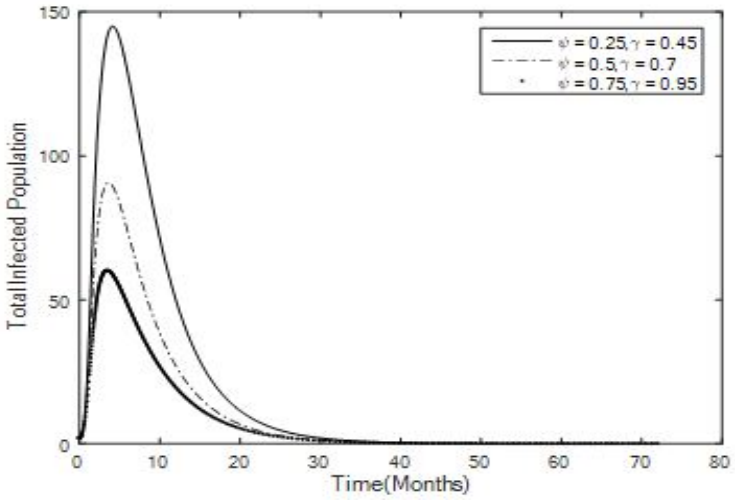


Figure 12: Effect of Testing and measles therapy and vaccination of susceptible humans without public enlighenment
$(\psi_1 = 0.25, \gamma_1 = 0.45); (\psi_2 = 0.5, \gamma_2 = 0.7); (\psi_3 = 0.75, \psi_3 = 0.95)$

All other parameters used are as given in Table 4.

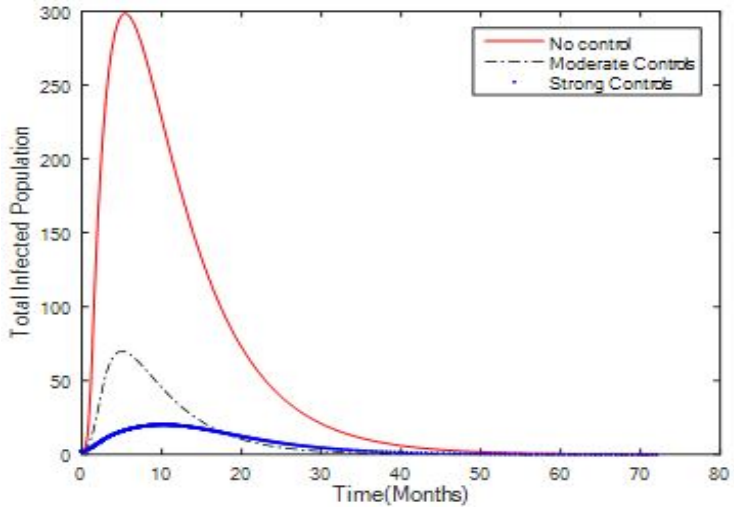


Figure 13: Graph of infected humans as a function of time with (1) no control (2) moderate control and (3) Strong control $(R_0 < 1)$

Parameters used are as given in Table 4.

6. Discussion of Results

In this section, the discussion is centered first on the results from the qualitative analysis of the model and then the results from the numerical simulations.
The qualitative analysis of the model shows that the solution of the model is bounded and positively invariant. We computed the basic reproduction number of the model R_0, using the next generation method given by

$$R_0 = \frac{(\alpha - \rho\alpha)[\theta\omega(\delta + \mu) + \delta\theta(1 - \omega)]}{(\varepsilon + \psi + \mu)(\tau + \kappa + \mu)[(\gamma + \mu)(\delta + \mu) - \delta\gamma]}$$

as a tool for effective disease management.
Stability analysis of the disease-free equilibrium state, DFEs, was explored using linearization method and taking R_0 as a threshold parameter. The results found in Theorem 1 and Theorem 2 shows that the disease-free equilibrium (DFE) state is locally as well as globally asympotically stable if the basic reproduction number is less than unity. The implication is that, measles can be eliminated from the population if the initial sizes of the populations of the model are in the basin of attraction of the DFE, (E_0) (Theorem 1). The results from Theorem 2 shows the DFE, (E_0) is globally asympotically stable. This implies that elimination of measles is independent of the initial sizes of the population.
Sensitivity analysis of R_0 with respect to: infection transmission rate (α), public enlightenment (ρ), vaccination of susceptible population (γ) and testing and measles therapy

(ψ) was carried out by the normalized forward sensitivity indices. The results of the sensitivity index of (R_0), is given in Table 3 and the assiociated graphs shown in Figure 2 – Figure 5.

The parameter, α has positive index, 1 as shown in Table 2 and the results from Figure 2 shows that basic reproduction number R_0, increases as the infection transmission parameter (α) value increases, it means that the average number of secondary cases of infectives increases in the population. It further shows that it has great impact on expanding (contracting) measles infection in the population if it value is increasing (decreasing).

The parameters ρ, γ, ψ, with negative sensitivity indices, $-1.500, -0.8062, -1.9607$, as shown respectively in Table 3 have an influence of minimizing the disease burden in the population as their values increases. Also as their values in Figure 2 – Figure 5 increases, the basic reproduction number decreases, which lead to minimizing the endemicity of the disease in the population.

Various numerical simulations are carried out to assess the feasibility of eradication of measles infection for different values of infection transmission, public enlightenment, vaccination, testing and measles therapy parameters.

The simulation in Figure 6 shows increasing prevalence of measles infection for increasing infection transmission rate $(\alpha = 0.006091; 0.03091; 0.8091)$ in the absence of any intervention $(\rho = 0, \gamma = 0, \psi = 0)$. With the basic reproduction number $R_0, < 1$, in each case, shows convergence of the solution profile to the disease-free equilibrium (DFE). This is consistent with Theorem 1 and Theorem 2. Thus the infection transmission rate plays a significant role as a preventive strategy.

The effect of single intervention strategy is displayed in Figure 7– Figure 9. The burden of measles infection in Figure7 shows slightly decreasing measles infection with increasing public enlightenment $(\rho = 0.04, 0.7, 0.9)$, while the infection transmission remain constant $(\alpha = 0, 09091)$. Figure8 display a substantive decline of measles burden by expanding or improving testing and measles therapy rate $(\psi = 0.25, 0.5, 0.75)$. Figure 9 also show a decreasing number of infected human by increasing the vaccination coverage of susceptible individuals $(\gamma = 0.4, 0.7, 0.95)$.

Figure 10–Figure 12 reveals the impact of combining two of the three intervention strategies. Figure 10 display a decreasing number of infected humans in the presence of increasing public enlightenment with testing and measles therapy rates $(\rho = 0.04, \psi = 0.25; \rho = 0.7, \psi = 0.5; \rho = 0.95, \psi = 0.75)$. The same decreasing number of infected human is noticed in Figure 11, with increasing public enlightenment and vaccination rates $(\rho = 0.04, \gamma = 0.45; \rho = 0.7, \gamma = 0.7; \rho = 0.9, \gamma = 0.95)$ and Figure 12, with increasing Testing and measles therapy with vaccination rates $(\psi = 0.25, \gamma = 0.45; \psi = 0.5, \gamma = 0.7; \psi = 0.75, \gamma = 0.95)$. The figure further revealed that an improve combination of testing and measles therapy with vaccination has a greater impact in reducing measles infection burden.

Figure 13 shows the dynamics of measles infection in the absence of any intervention strategies which is consistent with Figure 6, It further shows a decreasing number of infected individuals with increasing combination of the three control strategies. Thus we can deduce from Figure 13 that with a combination of mass public enlightenment campaigns, high testing and measles therapy with expanded vaccination coverage, measles infection can be eradicated in the shortest possible time.

CONCLUSION

A deterministic epidemiological model of measles infection incorporating three type of interventions strategies based on public enlightenment, vaccination of susceptible human with testing and measles therapy of exposed humans was formulated and analysed.
The qualitative analysis of the model shows that the solution is bounded and positively-invariant. The disease-free equilibrium (DFE) state of the model was determined and used to compute the basic reproduction number R_0, as a tool for effective disease management. Stability analysis for the disease-free equilibrium state (DFEs) was carried out and the results shows that it is locally as well as globally asymptotically stable whenever the basic reproduction number $R_0 < 1.R_04$ was analytically and numerically evaluated for its sensitivity to infection transmission rate, public enlightenment rate, vaccination rate with testing and measles therapy rate.
Numerical simulations of the model show that the infection transmission rate constitutes an essential key to preventive strategies. Numerical simulation of the model also revealed that effective vaccination or testing and measles therapy as a strategy, can eradicate measles infection. It is further shown that a combination of public enlightenment, vaccination with testing and measles therapy is vital to eradicating measles infection in the shortest possible time, thus these three control measures are key public health strategies to eliminating measles infection.

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REFERENCES

[1] AYOADE A., AGBOOLA S. & IBRAHIM M. (2019). Mathematical Analysis of the effect of maternal immunity on the global eradication of measles. *Annals of Computer Science Series*. **17**(1), 235–241.
[2] BOLARIN G. (2014). On the Dynamical Analysis of a new Model for Measles Infection. *International Journal of Mathematics Trends and Technology*. **7**, 144-155.

[3] CASTILLO-CHAVEZ C. FENG C. & HUANG W. (2002). On the computation of R_0 and its role on global stability. *J. Math. Biol.* **35** 1-22.

[4] CIDRAP (2019). *Centre for Infectious Disease Research and Policy*. Global measles outbreaks makes 2019 a record setting year. https://www.cidrap.umn.edu/news-perspective/2019/08/global-measles-outbreaks-make-2019-record-setting-year.

[5] CHITNIS N., CUSHING J. M. & HYMAN J. M. (2006). Bifurcation analysis of a mathematical model for malaria transmission. *SIAM Journal on Applied Mathematics*. **67**(1), 24-45.

[6] DIEKMANN O., HEESTERBEEK J. A. P. & METZ J. A. J. (1990). On the definition and the computation of the basic reproduction ratio R_0 in models for infectious diseases in heterogeneous populations. *J. Math. Biol.* **28**, 365-382.

[7] FINE P. E., EAMES K. & HEYMANN D. L. (2011). Herd immunity: a rough guide. *Clin. Infect. Dis.* **52**(7), 911-916.

[8] FRED M. O., SIGEY J. K., OKELLO J. A., OKWOYO J. M. & KANG'ETHE G. J. (2014). Mathematical modeling on the control of measles by vaccination: Case Study of KISII County, Kenya. *The SIJ Transactions on Computer Science Engineering and its Application (CSEA)*. **2**(3), 61-69.

[9] GRIFFIN D. (2016). The immune response in measles: virus control, clearance and protective immunity. *Viruses*. **8**(10), 282.

[10] HEFFERNAN J. M. (2005). Perspectives on the basic reproductive ratio. *J. R. Soc. Interface*. **2**, 281-293.

[11] KUPFERSCHMIDT K. (2011). Public health: Europe's embarrassing problem. *Science*. **336**(6080), 406-407. https://doi.org/10.1126/science.336.6080.406.;2012

[12] LAKSHMIKANTHAM S., LEELA S. & MARTYNYUK A. A. (1989). *Stability Analysis of Nonlinear Systems*. Marcel Dekker Inc., New York and Busel.

[13] LASSALE J. P. (1976). *The stability of dynamical systems*. SIAM, Philadelphia, Pa, USA.

[14] MEMON Z., QURESHI S. & MEMON B. R. (2020). Mathematical analysis for a new nonlinear measles epidemiological system using real incidence data from Pakistan. *The European Physical Journal Plus* **135**(378), 1-21.

[15] MOMOH A. A., IBRAHIM M. O., UWANTA J. I. & MANGA S. B. (2013). Mathematical model for control of measles epidemiology. *International Journal of Pure and Applied Mathematics*. **87**(5),707-718.

[16] OBUMNEKE I., ADAMU I. & ADO S. T. (2017). Mathematical model for the dynamics of measles under the combined effect of vaccination and measles therapy. *Int. J.Sci. and Tech.* **6**(6), 862-874.

[17] OCHOCHE J. M. & GWERYINA R. I. (2014). A mathematical Model of measles with vaccination and two phases of infectiousness. *IOSR J. of Maths*. **10**(1), 95-105.

[18] PETER O. J., AFOLABI O. A., VICTOR A. A., AKPAN C. E. & OGUNTOLU, F. A. (2018). Mathematical model for the control of measles. *J. Appl. Sci. Environ. Manage.* **22**(4), 571-576.

[19] ROBERTS L. (2015). Is measles next Science. **348**(6238), 958-963. https://doi.org/10.1126/science.348.6238.958 ; 2015.

[20] STEPHEN E., KITENGESO R. E., KIRIA G. T., FELICIAN N., MWEMA G. G. & MAFARASA A. P. (2015). A mathematical model for control and elimination of the transmission dynamics of measles. *Applied and Computational Mathematics*. **4**(6), 396-408.

[21] TAWHIR A. (2012). *Modelling and control of measles transmission in Ghana*. Master of Philosophy thesis. Kwame Nkrumah University of Science and Technology.

[22] UNICEF (2019). *Measles outbreaks continue unabated 5th Dec,2019*. https://www.unicef.org/press-releases/measles-outbreaks-continue-unabated-five-countries-accounted-nearly-half-all-measles.

[23] VAN DEN DRIESSCHE P. & WATMOUGH J. (2002). Reproduction numbers and sub-threshold endemic equilibra for compartmental models of disease transmission. *Mathematical Biosciences* **180**, 28-48.

[24] WHO (2019). *Measles–Global situation, Disease outbreak news*. https://www.who.int/csr/don/26-november-2019-measles-global_situation/en/.

[25] WU J., DHINGRA R., GAMBHIR M. & REMAIS J. V. (2013). Sensitivity analysis of infectious disease models: methods, advances and their application. *J. R. Soc Interface*. 10:201221018. http://dx.doi.org/10.1098/rsif.2012.1018.

[26] YANAGI Y., TAKEDA, M. & OHNO S. (2006). Measles virus: cellular receptors, tropism and pathogenesis. *J. Gen. Virol.* **87**(10), 2767-2779.