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Linear Programing Formulations for Optimum Students Transportation Need in a University with two Campuses

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Abstract

This work aimed at optimizing student's transportation need and that of the transport agencies at Federal University of Technology Minna, Nigeria, which operates two campuses. Simplex method was used as a method of solution by making use of Linear Program Solver (LiPS) Software. The results from the Linear Programing formulations showed percentage increments of 10%, 6%, 17%, and 3% students for the respective cases considered. The models which involve increasing round trip time is useful to the Management of Bus Park on the campus in making better decisions for optimal student transportation need.

1. Introduction

In various facets of life, it is important that people meet up with their appointments on time. Federal University of Technology (FUT) Minna transport authority in many cases is unable to convey the required number of students to and from both campuses for their timely activities, especially during test and examination periods when there are so many students to be conveyed to GidanKwano (GK) Campus from Bosso Campus. A good number of students reside in Bosso

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Campus or environs and rely on School Buses to convey them to GK for their timely activities. The distance between both campuses is 13km. It is important that students and staff who reside around one campus (usually Bosso Campus) and have some timely engagements in the other campus (usually GK Campus) should meet up with their activities on time. This often poses a problem due to various factors and it has been observed that the transport system in a significant number of cases is unable to convey the required number of students and other university staff to GK Campus at the time they are supposed to be there. Over time a lot of adjustments have been made and quite a number of vehicles have been added to the transportation system but the issue still persists. It is from this backdrop that the study sought to optimize student's transportation need and that of the transport agencies by formulating LP models that can handle the 5 identified cases confronting the management of the Bus Park in FUT Minna.
It was observed that amongst the many vehicles managed by the school, only a few are available and are put to use. Delayed boarding time in the bus park was also observed since the method of simultaneous boarding for long queues is not being carried out. These factors lead to a number of students being unsatisfied. The study identified that the management of Bus Park, FUT Minna is usually confronted with five (**??**) different cases which require better decisions that will help the students meet up with their timely activities especially at GK Campus. These 5 cases are explained below:

Case 1: Lecture Day with Moderately Populated Students in the Bus Park and with Limited Number of Buses
This is a case during lecture time where there are less or moderately populated students in the bus park. This could exist in periods when the semester is just beginning or when most lectures have been concluded towards the end of the semester. Such population of student could also exist on rare occasions at the middle of the semester and for so many other reasons. At the same time, when the Bus Park Management is faced with the issue of having few buses on ground, maybe due to problems like fuel scarcity, ailment of bus drivers, spoilt buses, and so on, the Bus Park Management might be left with a hard decision to make that will help satisfy the students, by ensuring they meet up with their timely activities.

Case 2: Lecture Days with Large Populated Students in the Bus Park and with Limited Number of Buses
This is a case during lecture time where there are large populated students in the bus park. This could exist in periods when a good number of students need to attend lecture for a general course such as Mat111 or GST110 or when applicants to the school (e.g. Interim Joint Matriculation Board (IJMB) students) need to be conveyed to GK Campus for electronic examination and many more reasons. At

the same time, when the Bus Park Management is faced with the issue of having limited buses on ground, maybe due to problems like ailment of bus drivers, spoilt buses, fuel scarcity and so on, the bus park management might be left with a hard decision to make in ensuring student satisfaction, having that they meet up with their timely activities.

Case 3: Examination Periods with Moderately Populated Students in the Bus Park and with Limited Number of Buses
In this case there are moderately populated students in the bus park. This could exist in periods when there is few or no general examination paper to be written for that particular day. At the same time, when the Bus Park Management is faced with the issue of having limited buses on ground, maybe due to problems like scarcity of fuel, ailment of bus drivers, faults with buses and so on, the bus park management might be left with a hard decision to make in ensuring student satisfaction, having them to meet up with their timely activities.

Case 4: Examination Periods with Large Populated Students in the Bus Park and with Limited Number of Buses
This is a case during examination periods where there is large population of students in the bus park. This could exist in periods when there are some general courses to be written and also due to several other reasons. At the same time, when the Bus Park Management is faced with the issue of having limited buses on ground, maybe due to problems like scarcity of fuel, ailment of bus drivers, spoilt buses and so on, the Bus Park Management might be left with a hard decision to make that will help satisfy the students, by ensuring they meet up with their timely activities.

Case 5: Ideal Case: All Vehicles Being Put to Use - Considering a Single Trip Only
This is an ideal case where all the vehicles managed by the school park are assumed to be put into use at the same time. This case is considered to determine the amount of passengers that can be conveyed in a single trip if all the vehicles managed by the bus park system are put into use. The amount of passengers determined will indicate whether at maximum population, all the vehicles are able to convey the students in a timely manner and also without hassles.
Linear programming (LP) has been applied to various transportation problems especially when there are different routes and it is paramount to find the best and shortest route to take based on certain constraints. The approach taken in this study has to do with minimizing time delays and conveying the optimal number of passengers or students were there is only one route involved and limited number of vehicles. Federal University of Technology Minna (FUTMinna) transport system is a venture operated under FUTMIN Ventures that handles inter-campus

transportation conveying students to and fro between GK Campus and Bosso campus. Although it's not the only transport system that shuttles between both campuses, it's the major transport system being used by students mainly because it operates from the campus and because there is a significant difference in transportation cost being much lower than other transportation systems that operate outside the campuses.

This study proposes a solution using linear programming that considers the timely activities (i.e. the time these staff and students are supposed to be in the other campus), the number of passengers (staff and students alike) that need to be conveyed to meet up with that time and the number of vehicles available especially at those periods where the number of students to be conveyed is overwhelming. Other factors such as boarding time and whether simultaneous boarding should be employed are also considered since there is a time delay during boarding of passengers on the vehicles.

Linear programming (LP, also called linear optimization) is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. Linear programming is a special case of mathematical programming (mathematical optimization).

More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. Its feasible region is a convex polytope, which is a set defined as the intersection of finitely many half spaces, each of which is defined by a linear inequality. Its objective function is a real-valued affine (linear) function defined on this polyhedron. A linear programming algorithm finds a point in the polyhedron where this function has the smallest (or largest) value if such a point exists.

Linear programming can be applied to various fields of study. It is widely used in mathematics, and to a lesser extent in business, economics, and for some engineering problems. Industries that use linear programming models include transportation, energy, telecommunications, and manufacturing. It has proven useful in modeling diverse types of problems in planning, routing, scheduling, assignment, and design (Nyor et al., 2014; Orchard-Hay, 1990; Ramamurthy, 2007; Schrijver, 1998; Sierksma, 2001).

2. Materials and Methods

A linear programming (LP) problem was formulated, presenting its general format with the use of LP Solver as a solution tool for solving linear, integer and goal programming problems. Data for this work were obtained from Primary sources by head count and personal interview with officials of FUT Minna Bus Park at Bosso campus in the months of February, March and May 2018. These

were done on the day of electronic examination (E-exam) for applicants seeking admission into FUT Minna, lecture days and examination periods.
It was observed that the number of times the vehicles stopped on the way while conveying passengers to GK campus in the morning hours was not significant since the vehicles normally get filled up before taking off from the park, having no reason to stop on the way. Hence, stopping time was not considered in the formulations.

Table 1: Data collected from the Bus Park during Lecture Days

Day	**Total Number of Students that Need to be Conveyed**	**Remaining Students at**			**Total Remaining Students**
		8am	**9am**	**10am**	
1 (Special case)	1125	168	88	30	286
2 (case 2)	1006	0	90	21	111
3 (case 1)	805	19	81	0	100

Remark 1: (i) The day of special case include lecture and pre-degree examination days; and (ii) Day 2 and 3 here represent case 2 and 1 respectively.

Table 2: Data Collected from the Bus Park During Examination Period

Day	**Total Number of Students that Need to be Conveyed**	**Remaining Students at 9:00am**
1 (case 4)	831	121
2 (case 3)	714	42

Remark 2: Day 1 and 2 here represent case 4 and 3 respectively.

Table 3: Vehicular Usage in FUT Minna Bus Park

S/N	Vehicle Type	Total Number of Vehicles	Carry -ing Capacity	Above carrying capacity	Time taken for a trip (min)	Loading time (min)	Alighting time (min)	Average observed available vehicles for	
								Lecture days(3hrs)	Exam Period (2hrs)
1	Marcopolo	2	54	81	30	20	7	2	2
2	14-14	2	60	91	35	20	7	2	2
3	Toyota Coaster	5	30	32	30	15	7	1	3
4	Toyota Hiace	6	18	18	20	10	5	2	5
5	Toyota Sienna	20	9	9	20	5	3	1	3
6	Mitsubishi L300	25	18	18	25	10	5	2	3
7	Volskwagen Sharon	9	6	9	20	5	3	1	1
8	Toyota	1	4	6	20	5	3	1	1

Remark 3: At the time of data collection, no vehicle was observed returning to Bosso campus after conveying students to GK campus in the morning.

Table 4: Expected Number of Round Trips Calculated Using Loading Time, Alighting Time, and Time Taken for a Trip

S/N	Vehicle Type	Time taken for a trip (min)	Loading time (min)	Alighting time (min)	Expected number of round-trips per vehicle	
					Lecture days (3hrs)	Exam period (2hrs)
1	Macopolo	30	20	7	2	1
2	14-14	35	20	7	2	1
3	Toyota Coaster	30	15	7	2	1
4	Toyota Hiace	20	10	5	3	2
5	Toyota Sienna	20	5	3	4	3
6	Mitsubishi L300	25	10	5	3	2
7	Volskwagen Sharon	20	5	3	4	3
8	Toyota	20	5	3	4	3

where the **Expected Number of Round Trip** is the number of times a vehicle with respect to its type should move from GK campus to Bosso campus, not necessarily carrying students, and back to GK campus with students being conveyed.

$$Expected\ Round\ Trip = \left\lfloor \frac{Total\ Round\ Trips\ Time}{Time\ For\ a\ Round\ Trip} \right\rfloor + 1$$

where:

$$Total\ Round\ Trips\ Time = Observation\ Time - Time\ For\ a\ Trip.$$

$$Time\ for\ a\ Round\ Trip = Boarding\ Time + 2\,(Time\ Taken\ for\ a\ Trip) + Alighting\ Time.$$

2.1. **Problem Formulation.** The five (**??**) different cases explained in section 1 are formulated in the sub-sections below:

CASE 1: This is an LP problem for FUT Minna transport system for maximizing the use of limited number of vehicles during lecture days with moderately populated students in the bus park. Maximize $Z = 54x_1 + 60x_2 + 30x_3 + 18x_4 + 9x_5 + 18x_6 + 6x_7 + 4x_8$
Subject to:

x_1									≤ 4
	x_2								≤ 4
		x_3							≤ 2
			x_4						≤ 6
				x_5					≤ 4
					x_6				≤ 6
						x_7			≤ 4
							x_8		≤ 4

i.e. x $\leq$ Average observed vehicle within 3hrs $\times$ Expected number of round trips per vehicle within 3hrs
where
Z = number of students the available vehicles can convey within this time frame (i.e 3hrs)
x = number of times each vehicle type should convey passengers from bosso to GK campus within 3hrs during lecture days without conveying passengers back to Bosso campus.

CASE 2: This is an LP problem for FUT Minna transport system for maximizing the use of limited number of vehicles during lecture days with very populated students in the bus park. Maximize $Z = 81x_1 + 91x_2 + 32x_3 + 18x_4 + 9x_5 + 18x_6 + 9x_7 + 6x_8$
Subject to:

x_1									≤ 4
	x_2								≤ 4
		x_3							≤ 2
			x_4						≤ 6
				x_5					≤ 4
					x_6				≤ 6
						x_7			≤ 4
							x_8		≤ 4

i.e. x $\leq$ Average observed vehicle within 3hrs $\times$ Optimized round trips within 3hrs

where
Z = number of students the available vehicles can convey within this time frame (i.e 3hrs)
x = number of times each vehicle type should convey passengers from bosso to GK campus within 3hrs during lecture days without conveying passengers back to Bosso campus.

CASE 3: This is an LP problem for FUT Minna transport system for maximizing the use of limited number of vehicles during exam periods with moderately populated students in the bus park. Maximize $Z = 54x_1 + 60x_2 + 30x_3 + 18x_4 + 9x_5 + 18x_6 + 6x_7 + 4x_8$
subject to:

x_1									≤ 2
	x_2								≤ 2
		x_3							≤ 3
			x_4						≤ 10
				x_5					≤ 9
					x_6				≤ 6
						x_7			≤ 3
							x_8		≤ 3

i.e x $\leq$ Average observed vehicle within 2hrs $\times$ Optimized round trips within 2hrs
where
Z = number of students the available vehicles can convey within this time frame (i.e 2hrs)
x = number of times each vehicle type should convey passengers from Bosso to GK campus within 2hrs during exam periods without conveying passengers back to Bosso campus

CASE 4: This is an LP problem for FUT Minna transport system for maximizing the use of limited number of vehicles during exam periods with very populated students in the bus park. Maximize $Z = 81x_1 + 91x_2 + 32x_3 + 18x_4 + 9x_5 + 18x_6 + 9x_7 + 6x_8$
subject to:

x_1									≤ 2
	x_2								≤ 2
		x_3							≤ 3
			x_4						≤ 10
				x_5					≤ 9
					x_6				≤ 6
						x_7			≤ 3
							x_8		≤ 3

i.e x $\leq$ Average observed vehicle within 2hrs $\times$ Optimized round trips within 2hrs
where
Z = number of students the available vehicles can convey within this time frame (i.e 2hrs)
x = number of times each vehicle type should convey passengers from bosso to GK campus within 2hrs during exam periods without conveying passengers back to Bosso campus

CASE 5: For the ideal case, this is an optimized LP model formulated for FUT Minna transport system when all the vehicles owned by the school are being put to use with the population being the maximum observed number of passengers in the bus park to be conveyed. Maximize $Z = 54x_1 + 60x_2 + 30x_3 + 18x_4 + 9x_5 + 18x_6 + 6x_7 + 4x_8$
Subject to:

x_1									≤ 2
	x_2								≤ 2
		x_3							≤ 5
			x_4						≤ 6
				x_5					≤ 20
					x_6				≤ 25
						x_7			≤ 9
							x_8		≤ 1

i.e x $\leq$ total number of each bus type
where
Z = number of students the vehicles can convey within either of the sampled time frame (i.e 2hrs or 3hrs)
x = number of times each vehicle type should convey passengers from Bosso to GK campus within either of the sampled time frame.

3. Results and Discussions

Case 1. The results from the LP Solver indicates optimal values of $x_{1,} x_{2,} x_{3,} x_{4,} x_{5,} x_6$, $x_{7,}$ and x_8 to be 4, 4, 2, 6, 4, 6, 4, and 4 respectively which implies that for FUT Minna Transport System to be able to optimize the use of limited vehicles so as to satisfy the students (a total number of 805 students for case 1), by ensuring they meet up with their timely activities, the few vehicles in the bus park with respect to its type i.e $x_{1,} x_{2,} x_{3,} x_{4,} x_{5,} x_{6,} x_{7,}$and x_8 should have a total number of 4, 4, 2, 6, 4, 6, 4, and 4 respective round trips within 3 hours that is during lecture days with moderately populated students, especially in the morning hours (having 8am, 9am, 10am lecture hours to be our sampled time). By making use of this model, a total number of 808 students (i.e 705 and more students if possible) will be conveyed within this time frame. Also, we have the reduced cost to be zero, which implies that if we increase or decrease the objective coefficients (i.e unit students) as many times as possible, the optimal solution will increase or decrease by zero (i.e the optimal solution remains 808 students).

Case 2. The results from the LP Solver indicates optimal values of $x_{1,} x_{2,} x_{3,} x_{4,} x_{5,} x_6$, $x_{7,}$ and x_8 to be 4, 4, 2, 6, 4, 6, 4, and 4 respectively which implies that for FUT Minna Transport System to be able to optimize the use of limited vehicles so as to satisfy the students (a total number of 1,006 students for case 2), by ensuring they meet up with their timely activities, the few vehicles in the bus park with respect to its type i.e $x_{1,} x_{2,} x_{3,} x_{4,} x_{5,} x_{6,} x_{7,}$and x_8 should have a total number of 4, 4, 2, 6, 4, 6, 4, and 4 respective round trips within 3 hours that is during lecture days with moderately populated students, especially in the morning hours (having 8am, 9am, 10am lecture hours to be our sampled time). By making use of this model, a total number of 1,064 students (i.e 895 and more students if possible) will be conveyed within this time frame. Also, we have the reduced cost to be zero, which implies that if we increase or decrease the objective coefficients (i.e unit students) as many times as possible, the optimal solution will increase or decrease by zero (i.e the optimal solution remains 1,064 students).

Case 3. The results from the LP Solver indicates optimal values of $x_{1,} x_{2,} x_{3,} x_{4,} x_{5,} x_6$ $x_{7,}$ and x_8 to be 2, 2, 3, 10, 9, 6, 3, and 3 respectively which implies that for FUT Minna Transport System to be able to optimize the use of limited vehicles so as to satisfy the students (a total number of 714 students for case 3), by ensuring they meet up with their timely activities, the few vehicles in the bus park with respect to its type i.e $x_{1,} x_{2,} x_{3,} x_{4,} x_{5,} x_{6,} x_{7,}$and x_8 should have a total number of 2, 2, 3, 10, 9, 6, 3, and 3 respective round trips within 2 hours that is during exam periods with moderately populated students, especially in the morning hours (having 9am exam hour to be our sampled time). By making use of this model, a total number of 717 students (i.e 672 and more students if possible) will be conveyed within this time frame. Also, we have the reduced cost to be zero, which implies that if we increase or decrease the objective coefficients (i.e unit students) as many times as possible, the optimal solution will increase or decrease by zero (i.e the optimal solution remains 717 students).

Case 4. The results from the LP Solver indicates optimal values of $x_1, x_2, x_3, x_4, x_5, x_6$, x_7, and x_8 to be 2, 2, 3, 10, 9, 6, 3, and 3 respectively which implies that for FUT Minna Transport System to be able to optimize the use of limited vehicles so as to satisfy the students (a total number of 831 students for case 4), by ensuring they meet up with their timely activities, the few vehicles in the bus park with respect to its type i.e $x_1, x_2, x_3, x_4, x_5, x_6, x_7$, and x_8 should have a total number of 2, 2, 3, 10, 9, 6, 3, and 3 respective round trips within 2 hours that is during exam periods with very populated students, especially in the morning hours (having 9am exam hour to be our sampled time). By making use of this model, a total number of 854 students (i.e 710 and more students if possible) will be conveyed within this time frame. Also, we have the reduced cost to be zero, which implies that if we increase or decrease the objective coefficients (i.e unit students) as many times as possible, the optimal solution will increase or decrease by zero (i.e the optimal solution remains 854 students).

Case 5. The results from the LP Solver indicates optimal values of $x_1, x_2, x_3, x_4, x_5, x_6$, x_7, and x_8 to be 2, 2, 5, 6, 20, 25, 9, and 1 trips respectively which implies that for an ideal case, by making use of all the vehicles managed by the Bus Park, would have ensured that the students meet up with their timely activities i.e $x_1, x_2, x_3, x_4, x_5, x_6, x_7$, and x_8 should have a total number of 2, 2, 5, 6, 20, 25, 9, and 1 trips meaning that if each x_j(j=1,2,3,4,5,6,7,8) go on a single trip to GK campus within 2 or 3 hours as the case may be, especially in the morning hours (i.e 8am, 9am, 10am lecture hours), a total number of 1,174 students can be conveyed within this time frame. This is more than the observed maximum population of passengers required to be conveyed within the time frame. Also, we have the reduced cost to be zero, which implies that if we increase or decrease the objective coefficients (i.e unit students) as many times as possible, the optimal solution will increase or decrease by zero (i.e the optimal solution remains 1,174 students).

Conclusion

The number of students being conveyed by FUT Minna bus park system was observed to be 705, 895, 672, and 710 for cases 1, 2, 3, and 4 respectively. But an implementation of the formulated models will yield the conveyance of 808, 1064, 717, and 854 students for cases 1,2,3, and 4 respectively. This means that our model yields additional 14%, 18%, 7%, and 20% for the respective cases.

The availability of vehicles in the bus park at any point in time is an important factor for optimal student transportation needed in FUT Minna. By implementing the formulated models as our recommendation, the management of the Bus Park can easily identify and efficiently plan out its transportation routine in order to meet the transportation need of the students. Further still, it will be very appropriate to consider the ideal case that yields the conveyance of 1174 students, which is above the observed number of students who needed to be conveyed to GK campus within our sampled time. In this ideal case (Case 5), all the vehicles owned by the transport system are put into use and without the need for multiple round trips per bus and all passengers required to be conveyed can be comfortably conveyed to GK campus and back to Bosso.

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