Nigerian Journal of Mathematics and Applications
Volume B, **29**, (2019), 129 − 146. *Printed by Unilorin press*

http://www.njmaman.com

Comparison between Taylor Series and Picard Method of Successive Approximation in Solving First Order Ordinary Differential Equation

S. A. Onitilo* and D. O. Daniel

Abstract

In this paper, Taylor series method and Picard's method of successive approximation is being compared with their exact solution of first order ordinary differential equation. For numerical analysis of the methods, four examples are considered. The results obtained are compared with their corresponding exact solutions. The results confirmed the effective and ease of using Picard's method of successive approximation and Taylor series method in solving first order ordinary differential equation (ODE).

1. Introduction

Many analytical, semi-analytical or purely numerical methods are available for the solution of differential equations encountered in management sciences, pure and applied sciences. Most of these methods are computationally intensive because they are trial-error in nature or need complicated symbolic computations (Mohamed, 2006)

The Taylor series algorithm is one of the earliest algorithms for the approximate solution for initial value problems for ordinary differential equations. It is a representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point (Ranjan, 1990; Rejeev, 2005; Dani, 2012). Newton used it in his calculation and Euler describe it in his

Received: 30/10/2019, Accepted: 12/12/2019, Revised: 20/12/2019. * Corresponding author.
2015 *Mathematics Subject Classification.* 35F10 & 35C10.
Key words and phrases. Taylor Series, Picard's method, ODE, First Order
Department of Mathematical Sciences, Olabisi Onabanjo University, Ago-Iwoye, Nigeria
Department of Mathematics and Computer Science, Southwestern University, Nigeria
E-mails: onitilo.sefiu@oouagoiwoye.edu.ng

work since then on can find many mention of it such as J. Liouville, G. Peano, E. Picard.

The Greek philosopher Zeno considered the problem of summing an infinite series to achieve a finite result, but rejected it as an impossibility (Lindberg, 2007), the result was Zeno's paradox. Later, Aristotle proposed a philosophical resolution of the paradox but the mathematical content was apparently unresolved until taken up by Democritus and then Archimedes. It was through Archimedes's method of exhaustion that an infinite number of progressive subdivisions could be performed to achieve a finite result (Kline, 1990).

Liu Hui independently employed a similar method a few centuries later (Boyer and Merzbach, 1991). In the 14th century, the earliest example of the use of Tayor series and closely related method were given by Madhava of Sangrama (Rejeev, 2005; Dani, 2012). Though no record of his work survives, writings of a late Indian Mathematicians suggest that he found a number of special cases of the Taylor series, including those for the trigonometric functions of sine, cosine, tangent and arctangent. Thekerala school of astronomy and mathematics further expanded his works with various series, expansions and rational approximation until 16th century.

In the 17th century, James Gregory also worked in this area and published several Maclaurin series. It was not until 1715 however that a general method for constructing these series for all function for which they exist was finally provided by Brook Taylor (Taylor, 1715), after whom the series are now named.

The Maclaurin series was named after Colin Maclaurin, a professor in Edinburgh, who published the special case of the Taylor result in the 18th century.

Picard's method of Successive approximation is a numerical method for solving differential equations. The concept of Picard method of successive approximation was first introduced by Rash (1987) used Adomian Decomposition method and Picard's method. Bellomo and Sarafyan (1987) compared Adomian Decomposition method with iterative scheme. Yossef (2007) used Picard's method of successive approximation with Gauss-seidel technique for initial value problem. Edeki *et al* examined the iterative techniques for numerical solutions of linear and nonlinear differential equations with consideration on differential transform method (DTM) and Picard's iterative method (PIM). They suggested suggested it's effective and reliable in obtaining approximate solution.

In this paper, the comparison of the numerical solution of first order ordinary differential equation using Taylor series and Picard's method of successive approximation with their exact solution is considered. Picard's method of successive approximation uses integration while Taylor series uses differentiation. Picard's iteration modifies and generate Taylor series. There is a conceptual connection between these two methods.

2. Material and Methods

In this section, analysis of basic methods which include basic concepts and theorems for Taylor series method and Picard's method of success approximation (or Picard's Iteration Method) are systematically introduced.

2.1. **Taylor Series Method.** The Taylor series method is one of the earliest method for the approximate solution for initial value problems for ordinary differential equations. It is representation of a function as an infinite sum of terms that are calculated from the value of functions derivatives at a single point. The Taylor series of a real or complex value function $f(x)$ that is infinitely differentiable at a real or complex number 'a' is the power series

$$f(a) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \frac{h^3}{3!}f'''(a) + \cdots + \frac{h^n}{n!}f^{(n)}(a)$$

where h is the interval i.e, $h = x - a$

Examples of Taylor series.

(1) Taylor series for the exponential function $exp(x)$ at a point "a" is

$$f(x) = e^x\left\{1 + (x-a) + \frac{(x-a)^2}{2!} + \frac{(x-a)^3}{3!} + \cdots\right\}$$

(2) Taylor series for $e^x \log(1+y)$ at $(a,b) = (0,0)$ is

$$f(x+y) = y + xy - \frac{y^2}{2!} + \cdots$$

The fundamental of Taylor Series Method: The following theorem called Taylor's theorem provides an estimate for the error function $E_n(x) = f(x) - P_n(x)$.

Theorem 2.1. *Let $f : [a,b] \to \mathbb{R}, f, f', f''', \ldots, f^{(n-1)}$ be continuous on $[x_0, x]$ and suppose $f^(n)$exist on (x_0, x). Then there exists $\xi \in (x, x_0)$ such that*

$$f(x) = f(x_0) + (x-x_0)f'(x_0,y_0) + \frac{(x-x_0)^2}{2!}f''(x_0,y_0)$$

$$+ \frac{(x-x_0)^3}{3!}f'''(x_0,y_0) + \frac{(x-x_0)^4}{4!}f^{iv}(x_0,y_0)$$

$$+ \frac{(x-x_0)^5}{5!}f^{v}(x_0,y_0) + \cdots + \frac{f^{(n-1)}(x_0)}{(n-1)!}(x-x_0)^{n-1}$$

$$+ \frac{f^{(n)}(\xi)}{n!}(x-x_0)^n.$$

2.2. **Picard's Method of Successive Approximation.** Consider the initial value problem of solving the first order differential equation $\frac{dy}{dx} = f(x,y)$ with the initial condition $y(x_0) = y0$. Recall that if f and $\partial f \partial y$ are both continuous on an open rectangle $I = (\alpha, \beta) \times (\gamma, \delta)$ and if $(x_0, y_0) \in I$ then for some interval $(x_{0-h}, x_{0+h}) \subset (\alpha, \beta)$ there exists a unique solution $y = \phi(x)$ to the initial value problem described above.

Assume that we want to solve the differential equation $\frac{dy}{dx} = f(x,y)$ with the initial condition $y(0) = 0$. This assumption will make the calculations that follow much simpler, and furthermore, we can always transform the differential equation
$\frac{dy}{dx} = f(x,y)$ with the initial condition $y(x_0) = y_0$ into a different differential equation with the initial condition $y(0) = 0$ using substitutions. The theorem for the existence of a unique solution $y = \phi(x)$ to this differential equation can be rephrased as such that if f and $\partial f \partial y$ are continuous on a rectangle $\mathbb{R}$ such that $-a \leq x \leq a$ and $-b \leq y \leq b$ then there exists an interval $(-h, h) \subset (-a, a)$ such that a unique solution $y = \phi(x)$ exists. Suppose that these conditions hold. Then let $y = \phi(x)$ be the unique solution. Then

$$\frac{dy}{dx} = f(x,y) = f(x, \phi(x)) \tag{1}$$

This is a function in terms of the variable t only. Suppose that we integrate this function starting at 0 to an arbitrary value of t. We then get that

$$\phi(t) = \int_0^x (s, \phi(s))ds \tag{2}$$

We should note that if $y = \phi(x)$ satisfies the above integral equation, then it also satisfies the initial value problem from earlier since by The Fundamental Theorem of Calculus we have that $d\phi dt = f(x, \phi(x)) = f(x,y)$ is a solution to the differential equation and $\phi(0) = \int_0^0 (s, \phi(s))ds = 0$ shows that $y = \phi(x)$ satisfies the initial condition.
Now consider the simplest function

$$\phi_0(x) = 0 \tag{3}$$

Clearly this function satisfies the initial condition (since $\phi_0(0) = 0$), though this function need not be a solution to our differential equation. So $\phi_0(x) = 0$ approximates the unique solution to our differential equation (though likely not that well). For a closer approximation to $y = \phi(x)$, consider the following functions obtained recursively

$$
\begin{aligned}
\phi_1(t) &= \int_0^x f(s, \phi_0(s))ds \\
\phi_2(t) &= \int_0^x f(s, \phi_1(s))ds \\
&\vdots \\
\phi_n(t) &= \int_0^x f(s, \phi_{n-1}(s))ds
\end{aligned} \tag{4}
$$

Notice that if for some $k = 0, 1, 2, \ldots$ we have that $\phi_k(x) = \phi_{k+1}(x)$, then $y = \phi_k(x)$ will be equal to the solution to our differential equation. This can easily be seen with some algebraic manipulation

$$
\begin{aligned}
\phi_k(x) = \phi_{k+1}(x) \Leftrightarrow \phi_k(x) \\
= \int_0^x f(s, \phi_k(s))ds \Leftrightarrow d\phi_k(x)dx \\
= f(x, \phi_k(x))
\end{aligned} \tag{5}
$$

Unfortunately, most of the time $\phi_k(x) \neq \phi_{k+1}(x)$ for any k. In such cases, we will want to consider the infinite sequence of functions ϕ_n that approximate the unique solution $y = \phi(x)$ to our initial value problem.

$$\{\phi_n\}_{n=0}^{n=\infty} = \{\phi_0, \phi_1, \phi_2, \ldots, \phi_n, \ldots\} \tag{6}$$

The following theorem says that the limit of this sequence of approximations will be equal to our unique solution.

Theorem 2.2. *If* $\dfrac{dy}{dx} = f(x, y)$ *is a first order differential equation with the initial condition* $y(0) = 0$ *and if* f *and* $\partial f, \partial y$ *are both continuous on some rectangle* $\mathbb{R}$ *for which* $-a \leq x \leq a$ *and* $-b \leq y \leq b$ *then*

$$
\begin{aligned}
\lim_{n\to\infty} \phi_n = \lim_{n\to\infty} \int_0^x f(s, \phi_{n-1}(s))ds \\
= \phi(x)
\end{aligned}
$$

where $y = \phi(x)$ *is the guaranteed unique solution contained in the interval* $(-h, h) \subset (-a, a)$.

Analysis of the Picard's Method of Successive Approximation (Edeki *et al*, 2014): Consider the first order ordinary differential equation (IVP)

$$y' = f(t, y), \qquad y(t_0) = y_0 \tag{7}$$

To guarantee the existence and uniqueness of the solution of (7), it is assumed that $f(t, y)$ is Lipschitz continuous in a ball $B_b^*(y_0)$; centre y_0 and radius b. We define a complete normed space $(H, < \cdot, \cdot > \| \cdot \|_f)$ for the function $f(t, y)$ equipped with the sup-norm:

$$\|f\| = \sup_{t \in [0,T]} |f(t, y(t))| \tag{8}$$

where H is a Hilbert space, $< \cdot, \cdot >$ an inner product, and $\| \cdot \|_v$ a norm operator with respect to v, such that:

$$f \in C[a, b] = \Lambda_a(t_0) \times B_b^*(y_0) \tag{9}$$

where

$$\Lambda_a(t_0) = [t_0 - a, t_0 + a] \text{ and } B_b^*(y_0) = [y_0 - b, y_0 + b] \tag{10}$$

Thus, for every pair of points (y_α, y_β) in Gr_f; the graph of f, there exists a constant $M > 0$, such that:

$$|f(t, y_\alpha) - f(t, y_\beta) \leq M|y_\alpha - y_\beta| \tag{11}$$

where M is a Lipschitz constant.
Now by integrating both sides of equation (7) we get

$$\int_{t_0}^{t} y'(\tau) d\tau = \int_{t_0}^{t} f(\tau, y(\tau)) d\tau \tag{12}$$

Thus, by fundamental theorem of calculus, (8) becomes

$$y(t) - y(t_0) = \int_{t_0}^{t} f(\tau, y(\tau)) d\tau$$

$$\therefore y(t) = y(t_0) + \int_{t_0}^{t} f(\tau, y(\tau)) d\tau \tag{13}$$

For an arbitrary t, it is obvious that $y(t)$ appears both on the left hand side and in the integrand of equation (13). Therefore, we have iterative approach (Picard's method of successive approximation) by choosing an initial guess $y(t_0) = 0$ and setting for $n \geq 1, n \in \mathbb{Z}^+$

$$y_{n+1}(t) = y_0 + \int_{t_0}^{t} f(\tau, y_n(\tau)) d\tau \tag{14}$$

Thus, the approximate solution to equation (7) is

$$y_{n+1}^{PM}(t) = y(t)$$

provided the limits in equation (14) exist such that

$$y_{n+1}^{PM}(t) = \lim_{n\to\infty} y_{n+1}(t) = \lim_{n\to\infty} y_n(t)y(t) \tag{15}$$

3. Numerical Simulation

In this section, we will consider some first order linear and non-linear ordinary differential equations (IVP) and solve them using both methods - the Taylor series method and the Picard's method of successive Approximation as discussed in the previous section and compared with their equivalent exact solution.

Application

In this section four examples of first order differential equations (IVP) are considered, their exact solution and numerical solution using the two methods - Taylor series and Picard's Method of Successive approximation.

Example 1 Consider the initial value problem (IVP)

$$y' = x^2 - y, \qquad y(0) = 1 \tag{16}$$

Exact Solution

$$y(x) = x^2 - 2x + 2 - e^{-x} \tag{17}$$

Taylor Series Method

$$y' = x^2 - y$$

$$y'' = 2x - y'; y''' = 2 - y''; y^{iv} = -y'''; y^{v} = -y^{iv}$$

The initial points are

$$y(0) = 1,\ y'(0) = -1,\ y''(0) = 1,\ y'''(0) = 1,\ y^{iv}(0) = -1,\ y^{v}(0) = 1$$

The fourth order Taylor's formula is

$$y(x) = y(x_0) + (x - x_0)y'(x_0, y_0) + \frac{(x - x_0)^2}{2!}y''(x_0, y_0)$$

$$+ \frac{(x - x_0)^3}{3!}y'''(x_0, y_0) + \frac{(x - x_0)^4}{4!}y^{iv}(x_0, y_0)$$

$$+ \frac{(x - x_0)^5}{5!}y^{v}(x_0, y_0)$$

hence

$$y(x) = 1 - x + \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{24}x^4 + \frac{1}{120}x^5 \tag{18}$$

Picard's Method of Successive Approximation: Starting with

$$y_0(x) = 1$$

we iterate

$$y_{k+1}(x) = 1 + \int_0^x (x^2 - (y_k(s))\, ds$$

so that

$$y_n(x) \to y(x)$$

$$y_1(x) = 1 + \int_0^x (s^2 - 1)\, ds$$

$$= 1 - x + \frac{x^3}{3}$$

$$y_2(x) = 1 + \int_0^x \left(s^2 - \left(1 - s + \frac{s^3}{3}\right)\right) ds$$

$$= 1 - x + \frac{x^2}{2} + \frac{x^3}{3} - \frac{s^4}{12}$$

$$y_3(x) = 1 + \int_0^x \left(s^2 - \left(1 - s + \frac{s^2}{2} + \frac{x^3}{3} - \frac{x^4}{12}\right)\right) ds$$

$$= 1 - x + \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \frac{x^5}{60}$$

After three iteration, we have the approximation

$$y(x) = 1 - x + \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \frac{1}{60}x^5 \tag{19}$$

Example 2. Consider the initial value problem (IVP)

$$y' - 2y = 3e^x, \qquad y(0) = 0 \tag{20}$$

Exact Solution

$$y(x) = -3(e^x - e^{2x}) \tag{21}$$

Taylor Series Method

$$y' = 3e^x + 2y; y'' = 3e^x + 2y'; y''' = 3e^x + 2y'';$$

$$y^{iv} = 3e^x + 2y'''; y^v = 3e^x + 2y^{iv}$$

The initial points are

$$y(0) = 0,\ y'(0) = 3,\ y''(0) = 9,\ y'''(0) = 21,\ y^{iv}(0) = 45,\ y^v(0) = 93$$

The fourth order Taylor's formula is

$$y(x) = y(x_0) + (x - x_0)y'(x_0, y_0) + \frac{(x - x_0)^2}{2!}y''(x_0, y_0)$$

$$+ \frac{(x - x_0)^3}{3!}y'''(x_0, y_0) + \frac{(x - x_0)^4}{4!}y^{iv}(x_0, y_0)$$

$$+ \frac{(x - x_0)^5}{5!}y^{v}(x_0, y_0)$$

hence

(22) $$y(x) = 3x + \frac{9}{2}x^2 + \frac{7}{2}x^3 + \frac{15}{8}x^4 + \frac{31}{40}x^5$$

Picard's Method of Successive Approximation: Starting with

$$y_0(x) = 0$$

we iterate

$$y_{k+1}(x) = \int_0^x (3e^x + 2y_k(s))\, ds$$

so that

$$y_n(x) \to y(x)$$

$$y_1(x) = \int_0^x (3e^s + 2(0)\, ds$$

$$= 3e^x - 3$$

$$y_2(x) = \int_0^x (3e^s + 2(3e^s - 3)\, ds$$

$$= 9e^x - 6x - 9$$

$$y_3(x) = \int_0^x (3e^s + 2(9e^s - 6s - 9)\, ds$$

$$= 21e^x - 6x^2 - 18x - 21$$

After three iteration, we have the approximation

(23) $$y(x) = 21e^x - 6x^2 - 18x - 21$$

Example 3. Consider the initial value problem (IVP)

$$y' = 3y + \cos x, \qquad y(0) = 0 \tag{24}$$

Exact Solution

$$y(x) = -\frac{3}{10}\cos x + \frac{1}{10}\sin x + \frac{3}{10}e^{-3x} \tag{25}$$

Taylor Series Method

$$y' = \cos x + 3y; y'' = -\sin x + 3y';$$

$$y''' = -\cos x + 3y''; y^{iv} = \sin x + 3y'y'';$$

$$y^{v} = \cos x + 3y^{iv}$$

The initial points are

$$y(0) = 0,\ y'(0) = 1,\ y''(0) = 3,\ y'''(0) = 8,$$

$$y^{iv}(0) = 24,\ y^{v}(0) = 73$$

The fourth order Taylor's formula is

$$y(x) = y(x_0) + (x - x_0)y'(x_0, y_0) + \frac{(x - x_0)^2}{2!}y''(x_0, y_0)$$

$$+ \frac{(x - x_0)^3}{3!}y'''(x_0, y_0) + \frac{(x - x_0)^4}{4!}y^{iv}(x_0, y_0)$$

$$+ \frac{(x - x_0)^5}{5!}y^{v}(x_0, y_0)$$

hence

$$y(x) = x + \frac{3}{2}x^2 + \frac{4}{3}x^3 + x^4 + \frac{73}{120}x^5 \tag{26}$$

Picard's Method of Successive Approximation: Starting with

$$y_0(x) = 0$$

we iterate

$$y_{k+1}(x) = \int_0^x (\cos s + 3(y_k(s)))\, ds$$

so that

$$y_n(x) \to y(x)$$

$$y_1(x) = \int_0^x (\cos s + 3(0))\, ds$$

$$= \sin x$$

$$y_2(x) = \int_0^x (\cos s + 3(\sin s))\, ds$$

$$= [\sin s - 3\cos s]_0^x$$

$$= \sin x - 3\cos x + 3$$

$$y_3(x) = \int_0^x (\cos s + 3(\sin s - 3\cos s + 3))\, ds$$

$$= -8\sin x - 3\cos x + 9x + 3$$

After three iteration, we have the approximation

(27) $$y(x) = -8\sin x - 3\cos x + 9x + 3$$

Example 4. Consider the initial value problem (IVP)

(28) $$y'(x) = 1 + y(x)^2, \qquad y(0) = 0$$

Exact Solution

(29) $$y(x) = \tan(x)$$

Taylor Series Method

$$y' = 1 + y^2$$

$$y'' = 2yy'; y''' = 2y'^2 + 2yy''; y^{iv} = 2yy''' + 6y'y'';$$

$$y^{v} = 2yy^{iv} + 8y'y''' + 6y''^2$$

The initial points are

$$y(0) = 0,\ y'(0) = 1,\ y''(0) = 0,\ y'''(0) = 2,$$

$$y^{iv}(0) = 0,\ y^{v}(0) = 16$$

The fourth order Taylor's formula is

$$y(x) = y(x_0) + (x - x_0)y'(x_0, y_0) + \frac{(x - x_0)^2}{2!}y''(x_0, y_0)$$

$$+ \frac{(x - x_0)^3}{3!}y'''(x_0, y_0) + \frac{(x - x_0)^4}{4!}y^{iv}(x_0, y_0)$$

$$+ \frac{(x - x_0)^5}{5!}y^{v}(x_0, y_0)$$

hence

$$y(x) = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 \tag{30}$$

Picard's Method of Successive Approximation: Starting with

$$y_0(x) = 0$$

we iterate

$$y_{k+1}(x) = \int_0^x (1 + (y_k(s))^2)\,ds$$

so that

$$y_n(x) \to y(x)$$

$$y_1(x) = \int_0^x (1 + 0^2)\,ds$$

$$= x$$

$$y_2(x) = \int_0^x (1 + s^2)\,ds$$

$$= x + \frac{x^3}{3}$$

$$y_3(x) = \int_0^x \left(1 + \left(s + \frac{s^3}{3}\right)^2\right)\,ds$$

$$= x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{x^7}{63}$$

After three iteration, we have the approximation

$$y(x) = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{1}{63}x^7 \tag{31}$$

Table 1: Numerical Comparison for Example 1

x	Exact Solution	Taylor	Picard	Absolute Error (Taylor)	Absolute Error (Picard)
0.0	1.0000000	1.0000000	1.0000000	0.0000000	0.0000000
0.1	0.9051625	0.9051625	0.9051585	0.0000000	4.0819640E-06
0.2	0.821269	0.821269	0.821205	8.64E-08	6.39E-05
0.3	0.749182	0.749183	0.748866	9.71E-07	0.000316279
0.4	0.689680	0.689685	0.688704	5.38E-06	0.000975954
0.5	0.643469	0.643490	0.641146	2.02E-05	0.002323507
0.6	0.611188	0.611248	0.606496	5.96E-05	0.004692364
0.7	0.593415	0.593563	0.584960	0.000148387	0.008455196
0.8	0.590671	0.590997	0.576661	0.000326297	0.014009703
0.9	0.603430	0.604083	0.581667	0.00065291	0.02176384
1.0	0.632121	0.633333	0.600000	0.001212775	0.032120559

Table 2: Numerical Comparison for Example 2

x	Exact Solution	Taylor	Picard	Absolute Error (Taylor)	Absolute Error (Picard)
0.0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.1	0.34869552	0.34869525	0.34858928	2.70E-07	0.000106241
0.2	0.811265818	0.811248	0.809457921	1.78E-05	0.001807897
0.3	1.416779978	1.41657075	1.407034959	0.000209228	0.009745019
0.4	2.201148693	2.199936	2.16831865	0.001212693	0.032830042
0.5	3.208681673	3.20390625	3.123146685	0.004775423	0.085534989
0.6	4.493994367	4.479264	4.304494808	0.014730367	0.189499559
0.7	6.124341778	6.08594175	5.748806857	0.038400028	0.375534921
0.8	8.182474488	8.093952	7.496359498	0.088522488	0.686114989
0.9	10.77013306	10.58431725	9.591665334	0.18581581	1.178467725
1.0	14.01232281	13.65	12.0839184	0.362322811	1.928404414

Table 3: Numerical Comparison for Example 3

x	Exact Solution	Taylor	Picard	Absolute Error (Taylor)	Absolute Error (Picard)
0.0	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
0.1	0.116439734	0.116439417	0.116320171	3.18E-07	0.000119563
0.2	0.2724826	0.272461333	0.27044562	2.13E-05	0.00203698
0.3	0.480832007	0.48057825	0.469828879	0.000253757	0.011003128
0.4	0.758658613	0.757162667	0.72147028	0.001495946	0.037188333
0.5	1.129174506	1.123177083	1.031848005	0.005997423	0.097326501
0.6	1.623757802	1.604904	1.406853368	0.018853802	0.216904434
0.7	2.284820086	2.234675917	1.85173194	0.05014417	0.433088146
0.8	3.16967651	3.051605333	2.371031145	0.118071177	0.798645366
0.9	4.355769218	4.10231475	2.968554818	0.253454468	1.3872144
1.0	5.947717484	5.441666667	3.647325204	0.506050817	2.30039228

Table 4: Numerical Comparison for Example 4

x	Exact Solution	Taylor	Picard	Absolute Error (Taylor)	Absolute Error (Picard)
0.0	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
0.1	0.100334672	0.100334667	0.100334668	5.42E-09	3.83E-09
0.2	0.202710036	0.202709333	0.202709537	7.02E-07	4.99E-07
0.3	0.30933625	0.309324	0.309327471	1.22E-05	8.78E-06
0.4	0.422793219	0.422698667	0.422724673	9.46E-05	6.85E-05
0.5	0.54630249	0.545833333	0.545957341	0.000469157	0.000345149
0.6	0.684136808	0.682368	0.682812343	0.001768808	0.001324465
0.7	0.84228838	0.836742667	0.838049878	0.005545714	0.004238503
0.8	1.029638557	1.014357333	1.017686146	0.015281224	0.011952411
0.9	1.260158218	1.221732	1.229324014	0.038426218	0.030834203
1.0	1.557407725	1.466666667	1.482539683	0.090741058	0.074868042

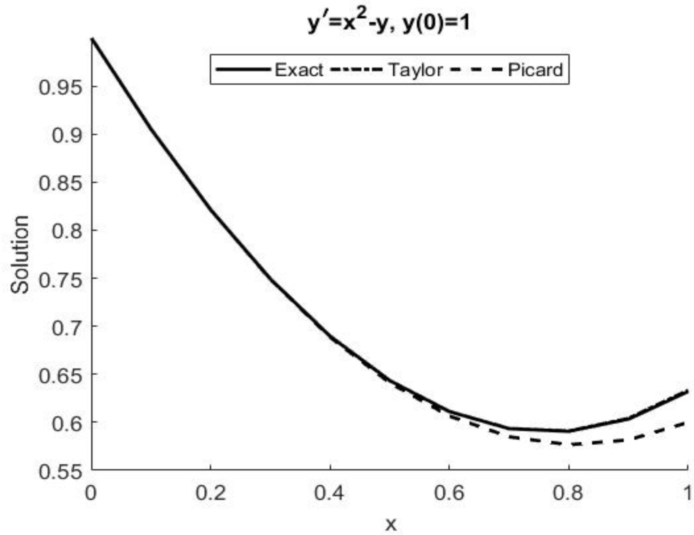

Figure 1: Graph of Example 1 Solutions

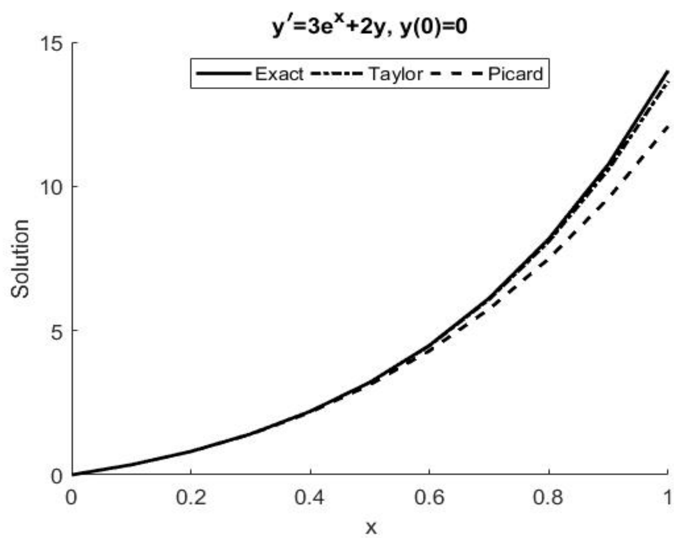

Figure 2: Graph of Example 2 Solutions

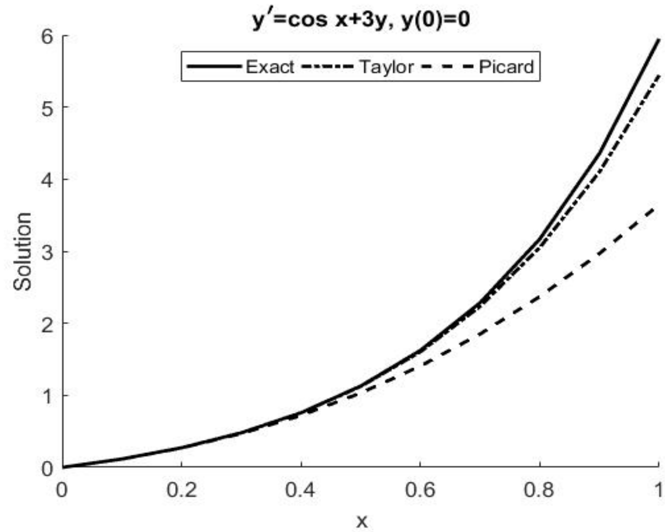

Figure 3: Graph of Example 3 Solutions

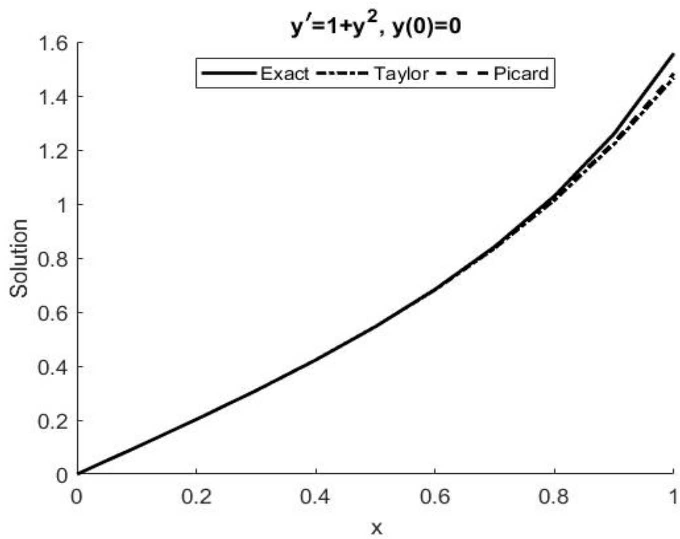

Figure 4: Graph of Example 4 Solutions

Discussion. Comparisons between the solution for each example are displayed in the above tables with their graph in figures 1-4 respectively. It is observed that results from Taylor series converge faster to their exact solution than results from Picard's Method.

Conclusion

In this paper, we have used Taylor series method and Picard's method of successive approximation successfully in solving first order linear and nonlinear differential equations (IVP), and the results obtained are computed, plotted and compared with their corresponding exact solutions.

It is observed and noted that Picard's iteration modifies and generate Taylor series. There is a conceptual connection between these two methods. More accuracy is recorded as the number of terms in the iteration increased. Results from Taylor series converge faster to their exact solution than results from Picard Method. The Taylor series method transform the differential equation to algebraic-recursive equations while the Picard's method of Successive Approximation transforms a differential equation to its equivalent in integral form provided the Lipschitz continuity condition is satisfied.

References

[1] Bellomo N. and Sarafyan D. (1987). On Adomian Decomposition method and some comparisons with Picard's method. J. Math. Anal. Appl. **123**, 389-400.

[2] Boyer C. and Merzbach U. (1991). A history of Mathematics (Second revised ed.). John Wiley and Sons. 202-203. ISBN 0-471-09763-2.

[3] Coddington E. A. and Levinson N. (1955). Theory of Ordinary Differential Equations. New York, McGraw-Hill.

[4] Dani S. G. (2012). Ancient Indian Mathematics A Conspectus. Resonance. **17** (3), 236246. doi:10.1007/s12045-012-0022-y.

[5] Edeki S. O., Opanuga A. A. and Okagbue, H. I. (2014). On iterative techniques for numerical solutions of linear and nonlinear differential equations. J. Math. Comput Sci. **4** (4), 716-727.

[6] Kline M. (1990). Mathematical Thought from Ancient to Modern Times. New York: Oxford University Press. 35-37, ISBN 0-19-506135-7

[7] Lindberg D. (2007). The Beginnings of Western Science (2nd ed.). University of Chicago Press. 33, ISBN 978-0-226-48205-7

[8] Rach R. (1987). On the Adomian Decomposition method and comparisons with Picard's method. J. Math. Anal. Appl. **128**, 480-483.

[9] Rajeev S. G. (2005). Neither Newton nor Leibniz: The Pre-History of Calculus in Medieval Kerala. A Colloquium and Several Lectures. Canisius College, Bufallo, NY. Archived (PDF) from the original on 2015-03-07. Retrieved 2019-09-16.

[10] Ranjan R. (1990, December). The Discovery of the Series Formula for π by Leibniz, Gregory and Nilakantha. Mathematics Magazine. **63** (5), 291-306.

[11] Taylor B. (1715). Methodus Incrementorum Directa et Inversa [Direct and Reverse Methods of Incrementation] (in Latin). London. 2123 (Prop. VII, Thm. 3, Cor. 2). Translated into

English in Struik, D. J. (1969). A Source Book in Mathematics 12001800. Cambridge, Massachusetts: Harvard University Press. 329-332.

[12] Yossef I. K. (2007). Picard iteration algorithm combined with Gauss-Seidel for initial value problem. Applied Mathematics and computation. **190**, 345-355.