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A Truncated Acceptance Sampling Inspection Plans for Finite and Infinite Lot Under Rayleigh Distribution

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Abstract

In this paper, single and double acceptance sampling plans were developed when the product life time follows a Rayleigh Distribution. The sample size (n) and Average Sample Number (ASN) for single and double sampling plans respectively were developed. The operating characteristic values for the two sampling plans were also obtained. The sampling plans were also obtained for various values of the parameters. It has been establish that, for a finite lot size, the sampling plans provide smaller values of the parameters to achieve the specified acceptance probabilities.

1. Introduction

Quality control is now one of the main tools that differentiate different commodities in the business market worldwide. Several important techniques for ensuring quality include acceptance sampling, design of experiment, statistical process control, product reliability control etc. Acceptance sampling inspection is a vital field in statistical quality assurance used to reject or accept products submitted for inspection. Product quality is a key feature for its acceptability. Products with high quality have higher acceptability compared with products with low quality. Quality control engineers ensure that the products supplied to

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the customers are of high quality and ensure that the market receives shipments that do not contain defective products. The quality supervisors are of the view that the manufactured products meet specific requirements during the manufacturing process. To meet the requirements, quality control engineers continuously monitor the production process and eventually the product quality. To make the process more efficient, quality control engineers set some rules for acceptance or rejection of the lot containing the products. In doing so, these engineers observe a specific number of items and fix a specific number of defective items for rejection of the lot; that is, if the number of defectives exceeds that fixed value, the lot is rejected. This is precisely the use of acceptance sampling in quality control. The basis for the acceptance sampling plans has been provided by [1]. Acceptance sampling plans have been discussed by several authors, such as [2]. The use of sampling plans in quality control has been discussed in detail by [3].

Particularly, products are classified on the basis of their life length, which is a random phenomenon and follows some probability model. Acceptance sampling plans have been studied by several authors, assuming that the life length of a product follows a specific probability distribution. [4] provided sampling plans that followed the assumption that the life length of a product follows the Weibull distribution. Acceptance sampling plans for the Gamma distribution has been constructed by [5]. Work on the construction of an acceptance sampling plan continued with the development of new probability models. In [6] sampling plans for the log-logistic distribution has been discussed. Reference [7] provided sampling plans that followed the assumption that the life length follows a generalized Rayleigh distribution. [8] constructed acceptance sampling plans when the life time of the product follows the Weibull distribution. The usefulness of sampling plans under various probability distributions depends upon the life length behavior of a product. The sampling plans discussed in this paper are useful when the life length of a product follows the Rayleigh Distribution. The double acceptance sampling plans are extensions of the single acceptance sampling plans. In these sampling plans, two samples are drawn for making decisions about acceptance or rejection of the lot on the basis of information obtained from the inspected samples. The double acceptance sampling plans have been discussed by [9]. This paper discusses single and double acceptance sampling plans when the life length of the component follows the Rayleigh distribution.

2. The Rayleigh Distribution

The Rayleigh distribution is a useful distribution to model lifetime data. The distribution was proposed and extensively studied by [10, 11, 12, 13]. The Rayleigh distribution is therefore widely applied in studies on reliability and survival analysis, engineering and communication technology. The Rayleigh distribution has also played a significant role in modeling the lifetime of random event. The density and cumulative distribution function (CDF) of a random variable following a Rayleigh distribution are:

$$f(t,\mu_0) = 2\frac{t}{\mu_0}e^{-(\frac{t}{\mu_0})^{\alpha}} \tag{1}$$

$$F\left(t,\ \mu_0\right) = 1-e^{-\frac{1}{2}(\frac{t}{\mu_0})^{\alpha}} \tag{2}$$

where μ_0 is the scale parameter (quality parameter or characteristics parameter) and α is the shape parameter. The expressions for mean and variance function for the distribution as given by [10] are:

$$E\left(X\right) = \frac{1}{2}\sqrt{\frac{\pi}{\mu}} \tag{3}$$

$$V\left(X\right) = \frac{4-\pi}{2}\sigma^2 \tag{4}$$

3. Acceptance Sampling Plans

Acceptance sampling plans are useful in quality control for acceptance or rejection of lot base on the information obtained from the lot slated for inspection [3]. Acceptance sampling plans provide a basis for deciding about acceptance or rejection of lots in manufacturing products. There are various sampling plans but the popular methods are single and double acceptance sampling plans. A single sampling plan involves determining the number of items to be inspected (n) and the maximum number of defective items among the inspected items (c) for acceptance of the lot. The single acceptance sampling plan is discussed by [2] and the acceptance probability of the lot is given as:

$$Pa\left(p\right) = \sum_{i=0}^{c}\binom{n}{i}p^i(1-p)^{n-\mathrm{i}} \tag{5}$$

where c is the acceptance number and p is some probability that an item will be defective during the testing time t or proportion of defective items in the lot. Single acceptance sampling plans for infinite lot size use Binomial distribution as the items can always be classified as good and defective.

Acceptance sampling plans are also describe by the life length of components being tested. Items are inspected over a specific time, and the experiment is terminated at a pre-fixed time t. The lot is accepted if fewer than or exactly c defective items occur in the time interval $[0,\ t]$. The acceptance or rejection of the lot is equivalent to testing of the hypothesis $H_0 : \mu > \mu_0$, where μ is life of the component and μ_0 is a pre-specified test value. During the construction of acceptance sampling plans, we do consider two important probabilities, namely, consumer risk (β) and producer risk (α). During the construction of single acceptance sampling plans, the values of n and c are obtained by solving the following two equations simultaneously for n and c:

$$\sum_{i=0}^{c} \binom{n}{i} (AQL)^i (1-AQL)^{n-i} \geq 1-\alpha \tag{6}$$

$$\sum_{i=0}^{c} \binom{n}{i} (LTPD)^i (1-LTPD)^{n-i} \leq 1-\beta \tag{7}$$

where AQL is Acceptable Quality Level and $LTPD$ is Lot Tolerance Percent Defective. Equations (6) and (7) use binomial distribution as it is assumed that the lot size is infinite.

The double acceptance sampling plan is an extension of the single sampling plan and entails drawing two samples. The algorithm for double acceptance sampling plan is described by [9] as below:

(1) Draw the first sample of size n_1 from a lot and put them on test until pre-fixed time t.
(2) Accept the lot if there is less or exactly c_1 or smaller number of failures. Reject the lot and terminate the test as soon as more than c_2 failures are observed. If the number of failures is between c_1 and c_2, then draw the second sample of size n_2 from the lot and put them on test until time t.
(3) Accept the lot if the total number of failures from the first and second samples is not greater than c_2. Otherwise, terminate the test and reject the lot.

The acceptance probability for a double sampling plan is given as:
Pa(p) = P(there no defect in sample 1) + P(1 defect occur in sample 1 and either 0 or 1 defect occur in sample 2) + P(2 defects occur in sample 1 and 0 defects occurs in sample 2).

$$\begin{cases} P_a(p) = \binom{n_1}{0} p^0 q^{n_1} + \binom{n_1}{1} p^1 q^{n_1-1} \left[\sum_{i=0}^{1} \binom{n_2}{i} p^i q^{n_2-i} \right] + \\ \binom{n_1}{2} p^2 q^{n_1-2} \left[\binom{n_2}{0} p^0 q^{n_2} \right] \end{cases} \tag{8}$$

The parameters of double acceptance sampling plan are determined by solving the following linear programing problem:
minimize ASN(LTPD), subject to: L(AQL)$\geq 1-\alpha$, L(LTPD) $\leq \beta$, $1 \leq n_2 \leq n_1$ and n_1 and n_2 where

$$ASN(p) = n_1 P_1 + (n_1 + n_2)(1 - P_1) \tag{9}$$

and P_1 is the probability that the lot is accepted on the basis of first sample, and is given as:

$$P_1 = 1 - \sum_{i=c_1+1}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1-i} \tag{10}$$

4. Single Acceptance Sampling Plans

This section developed a single sampling plan for life length of the components follows the Rayleigh distribution as given in (1). The sampling plan is obtained by considering two situations, namely, finite lot size and infinite lot size. The sampling plans are given in the following:

4.1. **Acceptance Sampling Plans for Infinite Lot Size.** The single acceptance sampling plan, in the case of an infinite lot size, is based upon obtaining the values of n and c, which satisfy (6) and (7), where AQL and $LTPD$ are probabilities obtained from the cumulative distribution function (CDF) of a Rayleigh distribution given in (2). To obtain the sampling plans, first consider the CDF and quantile function of power Lindley distribution as:

$$p = F(t, \mu_0) = 1 - e^{-\frac{1}{2}\left(\frac{t}{\mu_0}\right)^{\alpha}} \tag{11}$$

and

$$\begin{cases} F^{-1}(p; \mu_0^2) = \mu_0 \sqrt{2ln(p-1)} \\ = \mu_0 \sqrt{ln(1 - e^{-\frac{1}{2}\left(\frac{t}{\mu_0}\right)^{\alpha}} - 1)} \end{cases} \tag{12}$$

The acceptance sampling plans are constructed for various ratios $(\frac{\mu}{\mu_0})$, $\frac{t}{\mu_0}$, p and β. The values of n and c that satisfy (6) and (7). The different values of α, are obtain and given in the Tables. To obtain the values, it is assumed for $LTPD$ that $(\frac{\mu}{\mu_0}) = 1$. The values of n and c in these tables provide information about number of items to be put on test and number of defective items observed for rejection of the lot. For example, in Table A, the values of n and c for $p = 0.95, \beta = 0.05, \alpha = 0.01$, $a_0 = \frac{t}{\mu_0} = 0.5$ and $(\frac{\mu}{\mu_0}) = 3$ are 11 and 2 respectively. These values indicate that if the quality control engineer is interested in testing the hypothesis that the life length of a component is 1000hours and true average life is thrice this value, then the engineer can test 11 items; if fewer than or exactly 2 items are defective or fail during the testing period of 500hours; as $a_0 = 0.5$ and life length is in thousands of hours, then the engineer can conclude with 95% confidence that the life is more than 3000hours.

4.2. **Acceptance Sampling Plans for Finite Lot Size.** Often, the lot size from where the inspection is made is of finite size, say N; in this case, the acceptance plans given in the previous section do not work. In fact, (6) and (7) need suitable modification in this regard. It is a familiar fact that for a finite population size and under sampling without replacement, the hypergeometric distribution is an appropriate model to compute probabilities for specific characteristics of interest. Thus, the hypergeometric distribution is used instead of binomial probabilities in (6) and (7). Equations (6) and (7) for a finite lot size becomes:

$$\sum_{i=0}^{c} \frac{\binom{N+AQL}{i}\binom{N-N\times AQL}{n-i}}{\binom{N}{n}} \geq 1-\alpha \tag{13}$$

and

$$\sum_{i=0}^{c} \frac{\binom{N\times LTPD}{i}\binom{N-N\times LTPD}{n-i}}{\binom{N}{n}} \leq 1-\beta \tag{14}$$

where N is lot size. Then, using (13), the values of AQL and $LTPD$ can be obtained for various choices of $(\frac{\mu}{\mu_0})$ and various choices of parameters. The values of n and c that satisfy (13) and (14) are given in Tables C to D. The values of n and c in these tables are the number of items to be put on test and the number of defective items observed for rejection of the lot, respectively. For example, in Table C, the values of n and c for $N = 100$, $p = 0.95$, $\beta = 0.05$, $\alpha = 0.01$, $a_0 = 0.5$, and $(\frac{\mu}{\mu_0}) = 3$ are 10 and 2 respectively. These values indicate that if the quality control engineer is interested in testing the hypothesis that the life length of a component is 1000 h and true average life is thrice this value, then the engineer can test 10 out of 100 items; if fewer than 2 items fail in 500hours; as $a_0 = 0.5$ and life length is in thousands of hours, then the engineer can conclude with 95% confidence that the life is more than 3000 hours.

4.3. **Operating Characteristic Curves.** The operating characteristic curve is a useful way to assess the performance of an acceptance sampling plan. The operating characteristic values for a sampling plan provide the probability of acceptance of the lot under a given sampling plan when actual lot contains a specified percentage of defective items as given in (5). The operating characteristic values for the given sampling plan under the Rayleigh distribution with specific values of the parameters are calculated and given in Table A. It was observed that the probability of acceptance decreases as the value of $a_o = \frac{t}{\mu_0}$ increases for fixed ratio $(\frac{\mu}{\mu_0})$. Furthermore, it was also observed that for fixed value of $a_0 = \frac{t}{\mu_0}$ the acceptance probability increases as the ratio $(\frac{\mu}{\mu_0})$ increases.

4.4. **Double Acceptance Sampling Plans.** The double acceptance sampling plan is an extension of the single acceptance sampling plan and provides a way for acceptance or rejection of the lot by selecting two samples. The plan is based upon identifying values n_1, n_2, c_1 and c_2. The Average Sample Number (ASN) have been constructed for various choices of parameters and for $c_1 = 0$and $c_2=2$ and various ratios $(\frac{\mu}{\mu_0})$ when life length of the component follows the Rayleigh distribution. The values of n_1, n_2, for various values of parameters of the Rayleigh distribution are given in Table G.

Concluding Remarks

In this paper, acceptance sampling plans for when the life length of the component follows the Rayleigh distribution was discussed. The sampling plans have been constructed for various choices of the distribution parameters for single and double acceptance sampling plans. The single acceptance sampling plans have been constructed for finite and infinite lot sizes. It has also been noticed that the acceptance number decreases with an increase in the design parameters. It is also observed that with an increase in the ratio $(\frac{\mu}{\mu_0})$, the number of defective items required for acceptance of the lot decreases. Furthermore, it is also observed that the total number of items to be inspected is smaller for a finite lot size compared with the infinite lot size, and this difference decreases with an increase in the lot size. The sampling plans discussed in this paper are useful when the life length of components follow the Rayleigh distribution and other families of exponentiated distribution, and is always useful for quality control engineers who wants to decide about the acceptable life of the components. In such cases, the quality control engineer can use the plan parameters obtained in this paper for efficient decision making. The same observable fact has also been observed in the double acceptance sampling plans.

Appendix

Table A: Sample sizes for single sampling plan values of (n,c) for infinite lot size for various choices of parameters at $\alpha = 0.05$.

Table B: Sample sizes for single sampling plan values of (n,c) for infinite lot size for various choices of parameters at $\alpha = 0.01$.

Table C: Sample sizes for single sampling plan values of (n,c) for finite lot of size 100 using various choices of parameters and at $\alpha = 0.05$

Table D: Sample sizes for single sampling plan values of (n,c) for finite lot of size 300 using various choices of parameters and at $\alpha = 0.05$

Table E: Sample sizes for single sampling plan values of (n,c) for finite lot of size 500 using various choices of parameters and at $\alpha = 0.05$

Table A, Table B, Table C, Table D and Table E respectively are as follows:

a_0	p	β	$\frac{\mu}{\mu_0}$			
			2	3	4	5
0.5	0.75	0.01	(60,13)	(31,5)	(23,3)	(21,2)
		0.05	(41,9)	(21,4)	(19,2)	(14,2)
		0.10	(32,8)	(18,3)	(14,2)	(1322)
		0.25	(23,6)	(11,2)	(6,1)	(6,1)
	0.95	0.01	(31,12)	(17,5)	(13,3)	(10,2)
		0.05	(22,9)	(12,4)	(8,2)	(8,2)
		0.10	(19,8)	(9,3)	(7,2)	(5,1)
		0.25	(11,5)	(8,3)	(6,1)	(4,1)
1.0	0.75	0.01	(27,14)	(13,5)	(9,3)	(8,2)
		0.05	(17,9)	(9,4)	(6,3)	(6,2)
		0.10	(16,9)	(7,3)	(6,2)	(6,2)
		0.25	(10,6)	(6,3)	(5,2)	(3,1)
	0.95	0.01	(21,16)	(8,5)	(6,3)	(6,3)
		0.05	(14,11)	(6,4)	(5,3)	(4,2)
		0.10	(14,11)	(6,3)	(5,2)	(2,1)
		0.25	(11,9)	(4,3)	(3,1)	(2,1)
0.5	0.75	0.01	(58,8)	(33,3)	(27,2)	(21,1)
		0.05	(40,6)	(21,2)	(15,1)	(15,1)
		0.10	(31,5)	(18,2)	(13,1)	(13,1)
		0.25	(18,3)	(9,1)	(9,1)	(9,1)
	0.95	0.01	(27,7)	(17,3)	(14,2)	(11,1)
		0.05	(18,5)	(11,2)	(8,1)	(8,1)
		0.10	(17,5)	(9,2)	(7,1)	(7,1)
		0.25	(12,4)	(5,1)	(5,1)	(5,1)
1.0	0.75	0.01	(18,8)	(9,3)	(8,2)	(6,1)
		0.05	(13,6)	(6,2)	(6,2)	(5,1)
		0.10	(10,5)	(6,2)	(4,1)	(4,1)
		0.25	(7,4)	(3,1)	(3,1)	(3,1)
	0.95	0.01	(13,9)	(6,3)	(5,2)	(3,1)
		0.05	(11,8)	(5,3)	(4,2)	(3,1)
		0.10	(8,6)	(5,3)	(2,1)	(2,1)
		0.25	(5,4)	(3,2)	(2,1)	(2,1)
0.5	0.75	0.01	(25,7)	(15,3)	(13,2)	(10,1)
		0.05	(19,6)	(10,2)	(7,1)	(7,1)
		0.10	(15,5)	(9,2)	(6,1)	(6,1)
		0.25	(9,3)	(7,2)	(5,1)	(5,1)
	0.95	0.01	(16,8)	(8,3)	(7,2)	(5,1)
		0.05	(11,6)	(7,3)	(4,1)	(4,1)
		0.10	(9,5)	(5,2)	(4,1)	(4,1)
		0.25	(7,4)	(4,2)	(3,1)	(3,1)
1.0	0.75	0.01	(18,8)	(9,3)	(8,2)	(6,1)
		0.05	(13,6)	(6,2)	(6,2)	(5,1)
		0.10	(10,5)	(6,2)	(4,1)	(4,1)
		0.25	(7,1)	(3,1)	(3,1)	(3,1)
	0.95	0.01	(13,9)	(6,3)	(5,2)	(3,1)
		0.05	(11,8)	(5,3)	(4,2)	(3,1)
		0.10	(8,6)	(5,3)	(2,1)	(2,1)
		0.25	(5,4)	(3,2)	(2,1)	(2,1)

a_0	p	β	$\frac{\mu}{\mu_0}$			
			2	3	4	5
0.5	0.75	0.01	(81,19)	(43,8)	(32,5)	(28,4)
		0.05	(60,15)	(30,6)	(23,4)	(19,3)
		0.10	(46,12)	(24,5)	(17,3)	(17,3)
		0.25	(35,10)	(17,4)	(13,3)	(10,2)
	0.95	0.01	(45,19)	(21,7)	(17,5)	(15,4)
		0.05	(31,14)	(16,6)	(12,4)	(10,3)
		0.10	(26,12)	(13,5)	(9,3)	(9,3)
		0.25	(18,9)	(9,4)	(8,2)	(6,1)
1.0	0.75	0.01	(35,19)	(16,7)	(13,5)	(9,3)
		0.05	(26,15)	(13,6)	(9,4)	(8,3)
		0.10	(22,13)	(10,5)	(7,3)	(7,3)
		0.25	(16,10)	(7,4)	(6,3)	(5,2)
	0.95	0.01	(26,21)	(13,9)	(8,5)	(7,4)
		0.05	(23,19)	(11,8)	(6,4)	(5,3)
		0.10	(19,16)	(8,6)	(6,4)	(5,3)
		0.25	(15,13)	(5,4)	(4,3)	(3,2)
0.5	0.75	0.01	(71,11)	(43,5)	(33,3)	(27,2)
		0.05	(53,9)	(30,4)	(21,2)	(21,2)
		0.10	(44,8)	(22,3)	(18,2)	(18,2)
		0.25	(30,6)	(13,2)	(13,2)	(9,1)
	0.95	0.01	(37,11)	(19,4)	(17,3)	(14,2)
		0.05	(28,9)	(16,4)	(11,2)	(11,2)
		0.10	(24,8)	(12,3)	(9,2)	(9,2)
		0.25	(16,6)	(10,3)	(7,2)	(5,1)
1.0	0.75	0.01	(22,11)	(13,5)	(9,3)	(8,2)
		0.05	(17,9)	(9,4)	(6,2)	(6,2)
		0.10	(15,8)	(7,3)	(6,2)	(6,2)
		0.25	(10,6)	(6,3)	(5,2)	(3,1)
	0.95	0.01	(17,13)	(8,5)	(6,3)	(5,2)
		0.05	(14,11)	(6,4)	(5,3)	(4,2)
		0.10	(14,11)	(6,4)	(5,3)	(4,2)
		0.25	(11,9)	(4,3)	(3,2)	(3,2)
0.5		0.01	(30,7)	(18,3)	(15,2)	(12,1)
		0.05	(22,6)	(12,2)	(12,2)	(9,1)
		0.10	(18,5)	(10,2)	(7,1)	(7,1)
		0.25	(13,4)	(8,2)	(5,1)	(5,1)
		0.01	(16,7)	(10,3)	(8,2)	(6,1)
		0.05	(11,5)	(6,2)	(6,2)	(5,1)
		0.10	(10,5)	(6,2)	(4,1)	(4,1)
		0.25	(8,4)	(5,2)	(3,1)	(3,1)
1.0		0.01	(16,7)	(9,3)	(8,2)	(6,1)
		0.05	(13,6)	(6,2)	(6,2)	(5,1)
		0.10	(10,5)	(6,2)	(4,1)	(4,1)
		0.25	(7,4)	(5,2)	(3,1)	(3,1)
		0.01	(12,8)	(6,3)	(5,2)	(3,1)
		0.05	(10,7)	(5,3)	(4,2)	(3,1)
		0.10	(8,6)	(5,3)	(2,1)	(2,1)
		0.25	(5,4)	(3,2)	(2,1)	(2,1)

a_0	P	B	$\frac{\mu}{\mu_0}$			
			2	3	4	5
0.5	0.75	0.01	(39,8)	(26,4)	(19,2)	(19,2)
		0.05	(31,7)	(18,3)	(15,2)	(15,2)
		0.10	(26,6)	(16,3)	(13,2)	(9,1)
		0.25	(16,4)	(10,2)	(7,1)	(7,1)
	0.95	0.01	(26,10)	(14,4)	(12,3)	(10,2)
		0.05	(20,8)	(10,3)	(8,2)	(8,2)
		0.10	(17,7)	(9,3)	(7,2)	(5,1)
		0.25	(11,5)	(6,2)	(4,1)	(4,1)
1.0	0.75	0.01	(20,10)	(11,4)	(9,3)	(8,2)
		0.05	(15,8)	(9,4)	(6,2)	(4,1)
		0.10	(13,7)	(7,3)	(5,2)	(4,1)
		0.25	(10,6)	(6,3)	(5,2)	(3,1)
	0.95	0.01	(17,13)	(8,5)	(6,3)	(6,3)
		0.05	(14,11)	(6,4)	(5,3)	(4,2)
		0.10	(10,8)	(6,4)	(5,3)	(2,1)
		0.25	(10,8)	(4,3)	(3,2)	(2,1)
0.5	0.75	0.01	(38,5)	(24,2)	(19,1)	(19,1)
		0.05	(28,4)	(19,2)	(15,1)	(15,1)
		0.10	(25,4)	(17,2)	(12,1)	(12,1)
		0.25	(17,3)	(9,1)	(9,1)	(5,1)
	0.95	0.01	(23,6)	(13,2)	(10,1)	(10,1)
		0.05	(15,4)	(10,2)	(8,2)	(8,2)
		0.10	(14,4)	(9,2)	(7,2)	(7,2)
		0.25	(9,3)	(5,1)	(5,1)	(5,1)
1.0	0.75	0.01	(16,7)	(9,3)	(8,2)	(6,1)
		0.05	(11,5)	(6,2)	(4,1)	(4,1)
		0.10	(10,5)	(5,2)	(4,1)	(4,1)
		0.25	(7,4)	(3,1)	(3,1)	(2,1)
	0.95	0.01	(10,7)	(6,3)	(5,2)	(3,1)
		0.05	(8,6)	(5,3)	(4,2)	(3,1)
		0.10	(8,6)	(5,3)	(2,1)	(2,1)
		0.25	(5,4)	(3,2)	(2,1)	(2,1)
0.5		0.01	(20,4)	(14,2)	(11,1)	(11,1)
		0.05	(14,3)	(8,1)	(8,1)	(5,1)
		0.10	(12,3)	(7,1)	(7,1)	(4,1)
		0.25	(8,2)	(5,1)	(5,1)	(3,1)
		0.01	(11,4)	(8,2)	(6,1)	(6,1)
		0.05	(8,3)	(5,1)	(5,1)	(5,1)
		0.10	(7,3)	(4,1)	(4,1)	(2,1)
		0.25	(5,2)	(3,1)	(3,1)	(2,1)
1.0		0.01	(11,4)	(8,2)	(6,1)	(6,1)
		0.05	(8,3)	(4,1)	(4,1)	(4,1)
		0.10	(7,3)	(4,1)	(4,1)	(2,1)
		0.25	(6,3)	(3,1)	(2,1)	(2,1)
		0.01	(8,5)	(5,2)	(3,1)	(3,1)
		0.05	(6,4)	(4,2)	(3,1)	(3,1)
		0.10	(6,4)	(2,1)	(2,1)	(2,1)
		0.25	(3,2)	(2,1)	(2,1)	(2,1)

a_0	p	β	$\frac{\mu}{\mu_0}$			
			2	3	4	5
0.5	0.75	0.01	(48,10)	(31,5)	(24,3)	(20,2)
		0.05	(36,8)	(22,4)	(15,2)	(15,2)
		0.10	(30,7)	(17,3)	(13,2)	(10,1)
		0.25	(19,5)	(10,2)	(7,1)	(7,1)
	0.95	0.01	(31,12)	(17,5)	(13,3)	(10,2)
		0.05	(22,9)	(12,4)	(8,2)	(8,2)
		0.10	(19,8)	(9,3)	(7,2)	(5,1)
		0.25	(11,5)	(6,2)	(6,2)	(4,1)
1.0	0.75	0.01	(24,12)	(13,5)	(9,3)	(8,2)
		0.05	(17,9)	(9,4)	(6,2)	(6,2)
		0.10	(15,8)	(7,3)	(6,2)	(6,2)
		0.25	(10,6)	(6,3)	(5,2)	(3,1)
	0.95	0.01	(18,14)	(8,5)	(6,3)	(6,3)
		0.05	(14,11)	(6,4)	(5,3)	(4,2)
		0.10	(11,9)	(6,4)	(5,3)	(2,1)
		0.25	(11,9)	(4,3)	(3,2)	(2,1)
0.5	0.75	0.01	(46,6)	(31,3)	(26,2)	(21,1)
		0.05	(34,5)	(20,2)	(15,1)	(15,1)
		0.10	(26,4)	(17,2)	(13,1)	(13,1)
		0.25	(17,3)	(9,1)	(9,1)	(9,1)
	0.95	0.01	(27,7)	(16,3)	(13,2)	(10,1)
		0.05	(18,5)	(11,2)	(8,1)	(8,1)
		0.10	(14,4)	(9,2)	(7,1)	(7,1)
		0.25	(12,4)	(5,1)	(5,1)	(5,1)
1.0	0.75	0.01	(16,7)	(9,3)	(8,2)	(6,1)
		0.05	(13,6)	(6,2)	(4,1)	(4,1)
		0.10	(10,5)	(6,2)	(4,1)	(4,1)
		0.25	(7,4)	(3,1)	(3,1)	(2,1)
	0.95	0.01	(13,9)	(6,3)	(5,2)	(3,1)
		0.05	(11,8)	(5,3)	(4,2)	(3,1)
		0.10	(8,6)	(5,3)	(2,1)	(2,1)
		0.25	(5,4)	(3,2)	(2,1)	(2,1)
0.5		0.01	(20,4)	(15,2)	(11,1)	(11,1)
		0.05	(14,3)	(8,1)	(8,1)	(5,1)
		0.10	(13,3)	(7,1)	(7,1)	(4,1)
		0.25	(8,2)	(5,1)	(3,1)	(3,1)
		0.01	(11,4)	(8,2)	(6,2)	(6,1)
		0.05	(8,3)	(5,1)	(5,1)	(5,1)
		0.10	(7,3)	(4,1)	(4,1)	(2,1)
		0.25	(5,2)	(3,1)	(3,1)	(2,1)
1.0		0.01	(11,4)	(8,2)	(6,1)	(6,1)
		0.05	(9,4)	(4,1)	(4,1)	(4,1)
		0.10	(7,3)	(4,1)	(4,1)	(2,1)
		0.25	(6,3)	(3,1)	(2,1)	(2,1)
		0.01	(8,5)	(5,2)	(3,1)	(3,1)
		0.05	(6,4)	(4,2)	(3,1)	(3,1)
		0.10	(6,2)	(2,1)	(2,1)	(2,1)
		0.25	(3,2)	(2,1)	(2,1)	(2,1)

a_0	p	β	$\frac{\mu}{\mu_0}$			
			2	3	4	5
0.5	0.75	0.01	(56,12)	(32,5)	(24,3)	(20,2)
		0.05	(40,9)	(23,4)	(15,2)	(15,2)
		0.10	(30,7)	(17,3)	(13,2)	(10,1)
		0.25	(23,6)	(10,2)	(7,1)	(7,1)
	0.95	0.01	(31,12)	(17,5)	(13,3)	(10,2)
		0.05	(22,9)	(12,4)	(8,2)	(8,2)
		0.10	(19,8)	(9,3)	(7,2)	(5,1)
		0.25	(11,5)	(6,2)	(6,2)	(4,1)
1.0	0.75	0.01	(25,13)	(13,5)	(9,3)	(8,2)
		0.05	(17,9)	(9,4)	(6,2)	(6,2)
		0.10	(15,8)	(7,3)	(6,2)	(6,2)
		0.25	(10,6)	(6,3)	(5,2)	(3,1)
	0.95	0.01	(18,14)	(8,5)	(6,3)	(6,3)
		0.05	(14,11)	(6,4)	(5,3)	(4,2)
		0.10	(14,11)	(6,4)	(5,3)	(2,1)
		0.25	(11,9)	(4,3)	(3,2)	(2,1)
0.5	0.75	0.01	(51,7)	(32,3)	(27,2)	(21,1)
		0.05	(35,5)	(20,2)	(15,1)	(15,1)
		0.10	(27,4)	(18,2)	(13,1)	(13,1)
		0.25	(18,3)	(9,1)	(9,1)	(9,1)
	0.95	0.01	(27,7)	(16,3)	(14,2)	(11,1)
		0.05	(18,5)	(11,2)	(8,1)	(8,1)
		0.10	(14,4)	(9,2)	(7,1)	(7,1)
		0.25	(12,4)	(5,1)	(5,1)	(5,1)
1.0	0.75	0.01	(16,7)	(9,3)	(8,2)	(6,1)
		0.05	(13,6)	(6,2)	(6,2)	(5,1)
		0.10	(10,5)	(6,2)	(4,1)	(4,1)
		0.25	(7,4)	(3,1)	(3,1)	(2,1)
	0.95	0.01	(13,9)	(6,3)	(5,2)	(3,1)
		0.05	(11,8)	(5,3)	(4,2)	(3,1)
		0.10	(8,6)	(5,3)	(2,1)	(2,1)
		0.25	(5,4)	(3,2)	(2,1)	(2,1)
0.5		0.01	(21,4)	(15,2)	(11,1)	(11,1)
		0.05	(14,3)	(8,1)	(8,1)	(5,1)
		0.10	(13,3)	(7,1)	(7,1)	(4,1)
		0.25	(8,2)	(5,1)	(5,1)	(3,1)
		0.01	(11,4)	(8,2)	(6,2)	(6,2)
		0.05	(8,3)	(5,1)	(5,1)	(5,1)
		0.10	(7,3)	(4,1)	(4,1)	(2,1)
		0.25	(6,3)	(3,1)	(3,1)	(2,1)
1.0		0.01	(11,4)	(8,2)	(6,1)	(6,1)
		0.05	(9,4)	(5,1)	(5,1)	(5,1)
		0.10	(7,3)	(4,1)	(4,1)	(2,1)
		0.25	(6,3)	(3,1)	(2,1)	(2,1)
		0.01	(8,5)	(5,2)	(3,1)	(3,1)
		0.05	(6,4)	(4,2)	(3,1)	(3,1)
		0.10	(6,4)	(2,1)	(2,1)	(2,1)
		0.25	(3,2)	(2,1)	(2,1)	(2,1)

Table F1: Operating characteristic values for single sampling plan for Rayleigh sampling plan at $\boldsymbol{\alpha = 1.5}$

		$\alpha = 1.5$; $p = 0.85$; $c = 2$						$\alpha = 1.5$; $p = 0.90$; $c = 2$			
n	**A**	$\frac{\mu}{\mu_0}$				**N**	**a**	$\frac{\mu}{\mu_0}$			
		2	**3**	**4**	**5**			**2**	**3**	**4**	**5**
16	0.4	0.7962	0.9534	0.9855	0.9944	16	0.4	0.7163	0.9299	0.9776	0.9912
12	0.6	0.6008	0.8933	0.9653	0.9863	12	0.6	0.4812	0.8438	0.9470	0.9787
10	0.8	0.3978	0.8046	0.9326	0.9728	10	0.8	0.2731	0.7242	0.8987	0.9579
9	1.0	0.2166	0.6784	0.8786	0.9492	9	1.0	0.1212	0.5677	0.8222	0.9226
7	1.2	0.1941	0.6597	0.8719	0.9470	7	1.2	0.1049	0.5444	0.8118	0.9187
6	1.4	0.1471	0.6062	0.8465	0.9359	6	1.4	0.0734	0.4833	0.7766	0.9020
5	1.6	0.1437	0.6000	0.8442	0.9355	5	1.6	0.0720	0.4763	0.7729	0.9009
		$\alpha = 1.5$; $p = 0.95$; $c = 2$						$\alpha = 1.5$; $p = 0.99$; $c = 2$			
n	**A**	$\frac{\mu}{\mu_0}$				**N**	**a**	$\frac{\mu}{\mu_0}$			
		2	**3**	**4**	**5**			**2**	**3**	**4**	**5**
12	0.4	0.7467	0.9411	0.9818	0.9929	12	0.4	0.5057	0.8548	0.9512	0.9804
10	0.6	0.4433	0.8284	0.9419	0.9767	10	0.6	0.1729	0.6321	0.8551	0.9379
9	0.8	0.1895	0.6518	0.8659	0.9433	9	0.8	0.0347	0.3748	0.6978	0.8573
7	1.0	0.1332	0.5875	0.8354	0.9301	7	1.0	0.0186	0.2999	0.6390	0.8252
6	1.2	0.0788	0.4948	0.7838	0.9056	6	1.2	0.0077	0.2122	0.5514	0.7710
5	1.4	0.0663	0.4627	0.7642	0.8965	5	1.4	0.0061	0.1873	0.5203	0.7504
4	1.6	0.0921	0.5082	0.7922	0.9111	4	1.6	0.0117	0.2286	0.5638	0.7796

Table F2: Operating characteristic values for single sampling plan for Rayleigh sampling plan at $\boldsymbol{\alpha = 2.0}$

		$\boldsymbol{\alpha = 2.0; \; p = 0.85; \; c = 2}$						$\boldsymbol{\alpha = 2.0; \; p = 0.90; \; c = 2}$			
$\boldsymbol{N}$	$\boldsymbol{A}$	$\frac{\mu}{\mu_0}$				$\boldsymbol{N}$	$\boldsymbol{a}$	$\frac{\mu}{\mu_0}$			
		2	**3**	**4**	**5**			**2**	**3**	**4**	**5**
16	0.4	0.9284	0.9910	0.9995	16	0.4	0.8931	0.9992	16	0.4	0.9284
12	0.6	0.7856	0.9662	0.9979	12	0.6	0.7025	0.9965	12	0.6	0.7856
10	0.8	0.5815	0.9157	0.9939	10	0.8	0.4634	0.9902	10	0.8	0.5815
9	1.0	0.3523	0.8254	0.9852	9	1.0	0.2363	0.9766	9	1.0	0.3523
7	1.2	0.2922	0.7937	0.9819	7	1.2	0.1839	0.9714	7	1.2	0.2922
6	1.4	0.2085	0.7349	0.9748	6	1.4	0.1180	0.9606	6	1.4	0.2085
5	1.6	0.1814	0.7115	0.9720	5	1.6	0.0986	0.9562	5	1.6	0.1814
		$\boldsymbol{\alpha = 2.0; \; p = 0.95; \; c = 2}$						$\boldsymbol{\alpha = 2.0; \; p = 0.99; \; c = 2}$			
$\boldsymbol{N}$	$\boldsymbol{A}$	$\frac{\mu}{\mu_0}$				$\boldsymbol{n}$	$\boldsymbol{a}$	$\frac{\mu}{\mu_0}$			
		2	**3**	**4**	**5**			**2**	**3**	**4**	**5**
12	0.4	0.9087	0.9881	0.9993	12	0.4	0.7907	12	0.4	0.9087	0.9881
10	0.6	0.6803	0.9429	0.9961	10	0.6	0.4293	10	0.6	0.6803	0.9429
9	0.8	0.3767	0.8377	0.9865	9	0.8	0.1381	9	0.8	0.3767	0.8377
7	1.0	0.2613	0.7736	0.9795	7	1.0	0.0702	7	1.0	0.2613	0.7736
6	1.2	0.1517	0.6800	0.9671	6	1.2	0.0268	6	1.2	0.1517	0.6800
5	1.4	0.1109	0.6283	0.9594	5	1.4	0.0158	5	1.4	0.1109	0.6283
4	1.6	0.1244	0.6443	0.9625	4	1.6	0.0202	4	1.6	0.1244	0.6443

Table G: Average Sample Number (ASN) for Double Sampling plan at c_1 =0 and $c_2 = 2$

β	$\frac{t}{\mu_0}$	n_1	n_2	$\frac{\mu}{\mu_0} = 2$		$\frac{\mu}{\mu_0} = 4$		$\frac{\mu}{\mu_0} = 6$	
				Pa(p)	ASN	Pa(p)	ASN	Pa(p)	ASN
0.25	0.628	3	5	0.9178	7	0.9850	8	0.9949	8
	0.912	3	5	0.8191	7	0.9613	8	0.9862	8
	1.257	3	5	0.2592	2	0.9176	7	0.9686	8
	1.571	3	5	0.5501	4	0.8662	7	0.9460	8
	2.356	2	4	0.5238	3	0.8522	5	0.9392	6
	3.141	2	4	0.3446	2	0.7445	4	0.8844	5
	3.927	2	4	0.2189	1	0.6314	4	0.8177	5
	4.712	2	3	0.2051	1	0.6292	3	0.8183	4
0.10	0.628	4	7	0.8293	9	0.9639	11	0.9872	11
	0.912	3	6	0.7802	7	0.9500	9	0.9817	9
	1.257	3	4	0.3250	2	0.9386	7	0.9773	7
	1.571	3	4	0.6222	4	0.8981	6	0.9604	7
	2.356	3	4	0.3662	3	0.7680	5	0.8981	6
	3.141	2	4	0.3446	2	0.7445	4	0.8844	5
	3.927	2	4	0.2189	1	0.6314	4	0.8177	5
	4.712	2	3	0.2051	1	0.6292	3	0.8183	4
0.05	0.628	5	6	0.8128	9	0.9600	11	0.9858	11
	0.912	5	6	0.6377	7	0.9043	10	0.9631	11
	1.257	5	6	0.0780	1	0.8125	9	0.9209	10
	1.571	4	5	0.4394	4	0.8132	7	0.9211	8
	2.356	4	5	0.1942	2	0.6199	6	0.8133	7
	3.141	4	5	0.0789	1	0.4397	4	0.6852	6
	3.927	3	4	0.1011	1	0.4845	3	0.7197	5
	4.712	3	4	0.0508	0	0.3662	3	0.6223	4
0.01	0.628	5	6	0.8128	9	0.9600	11	0.9858	11
	0.912	5	6	0.6377	7	0.9043	10	0.9631	11
	1.257	5	5	0.1013	1	0.8431	8	0.9357	9
	1.571	5	5	0.3472	3	0.7581	8	0.8934	9
	2.356	4	5	0.1942	2	0.6199	6	0.8133	7
	3.141	4	5	0.0789	1	0.4397	4	0.6852	6
	3.927	3	4	0.1011	1	0.4845	3	0.7197	5
	4.712	3	4	0.0508	0	0.3662	3	0.6223	4

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