Nigerian Journal of Mathematics and Applications
Volume **27**, (2018), 89 − 98. *Printed by Unilorin press*
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An Operator for Obtaining Nth Order Derivative of a Function Depending on Seven Independent Variables

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Abstract

Leibnitz's theorem is used in the product rule to generate a series method for computing the nthorder derivative of a function that depends on seven Independent variables. The new series method does not require the knowledge of the preceding derivative before obtaining the succeeding ones. A generalization of the method is also presented.

1. Introduction

An operator for obtaining the N^{th} order derivative of the function:

$$y=\prod_{i=1}^{7}L_i,\ y=\prod_{i=1}^{7}\frac{1}{L_i},\ y=\prod_{\substack{i=1\\ m=6}}^{\substack{m=7\\ i=5}}\frac{L_i}{L_m},\ y=\prod_{\substack{i=1\\ m=5}}^{\substack{m=7\\ i=4}}\frac{L_i}{L_m},\ y=\prod_{\substack{i=1\\ m=4}}^{\substack{m=7\\ i=3}}\frac{L_i}{L_m}$$

Received: 01/07/2018, Accepted: 10/12/2018, Revised: 30/12/2018.
2015 *Mathematics Subject Classification.* 90C56, & 40C15.
Key words and phrases. Product rule; Leibnitz's Theorem; Higher Order Derivatives; Mathematical Induction
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or

$$\boldsymbol{y = \prod_{\substack{i=1\\m=3}}^{\substack{m=7\\i=2}} \frac{L_i}{L_m}}$$

is investigated.
The interest is to derive a method that makes it easy to obtain the n^{th} order derivative of the function of the type:

$$\mathcal{Y} = \prod_{i=1}^{7} \ell_i(x) = \prod_{i=1}^{3} \ell_1(x) \prod_{m=4}^{7} \ell_m(x) \tag{1}$$

This approach is expected to work for functions of the forms:

$$\frac{\prod_{i=1}^{3} \ell_i(x)}{\prod_{m=4}^{7} \ell_m(x)} \tag{2}$$

It is good to note that the n^{th} order derivative of

$$\frac{\prod_{i=1}^{4} \ell_i(x)}{\prod_{m=5}^{7} \ell_m(x)} \tag{3}$$

can be obtained by same method and this will also work for the n^{th} order derivative of

$$\frac{\prod_{i=1}^{2} \ell_i(x)}{\prod_{m=3}^{7} \ell_m(x)} \tag{4}$$

At the end, we shall be able to give a generalized operator for any of (1), (2), (3) and (4) by mathematical induction. See [4, 5, 6 & 7] for other methods.

2. Main Result

Our take off point is to invoke the product rule on (1) such that $\mathcal{Y}$ is expressed as a product of two sets of functions.
Thus

$$\mathcal{Y} = \mathcal{U}(x)\,\mathcal{V}(x) \tag{5}$$

where

$$\mathcal{U}(x) = \prod_{i=1}^{3} \ell_i(x) \tag{6}$$

and

$$\mathcal{V}(x) = \prod_{m=4}^{7} \ell_m(x) \tag{7}$$

By these, the first derivative of (5) is given as:

$$\frac{dy}{dx} = \mathcal{U}(x)\frac{dv}{dx} + \mathcal{V}(x)\frac{du}{dx} \tag{8}$$

such that

$$\frac{dy}{dx} = \frac{d^0}{dx^0}\left[\prod_{i=1}^{3} \ell_i(x)\right]\frac{d}{dx}\left[\prod_{i=4}^{7} \ell_i(x)\right] + \frac{d^0}{dx^0}\left[\prod_{i=4}^{7} \ell_i(x)\right]\frac{d}{dx}\left[\prod_{i=1}^{3} \ell_i(x)\right] \tag{9}$$

The work in [1, 2, 3] obtained the n^{th} order derivatives of the functions of the forms:

$$\mathcal{Y} = \prod_{i=1}^{3} \ell_i(x) \tag{10}$$

and

$$\mathcal{Y} = \prod_{m=1}^{4} \ell_m(x) \tag{11}$$

by differentiating (9), we get:

$$\begin{aligned} \frac{d^2y}{dx^2} = \prod_{i=1}^{3} \ell_{?.}(x)\frac{d^2}{dx^2}\left[\prod_{i=4}^{7} \ell_i(x)\right] + \frac{d}{dx}\left[\prod_{i=1}^{3} \ell_i(x)\right]\frac{d}{dx}\left[\prod_{i=4}^{7} \ell_i(x)\right] \\ + \frac{d^2}{dx^2}\left[\prod_{i=1}^{3} \ell_i(x)\right]\prod_{i=4}^{7} \ell_i(x) + \frac{d}{dx}\left[\prod_{i=4}^{7} \ell_i(x)\right]\frac{d}{dx}\left[\prod_{i=1}^{3} \ell_i(x)\right] \end{aligned} \tag{12}$$

such that

(13)

$$\frac{d^2y}{dx^2} = \frac{d^0}{dx^0}\prod_{i=1}^{3} \ell_i(x)\frac{d^2}{dx^2}\left[\prod_{i=4}^{7} \ell_i(x)\right] + 2\left[\frac{d}{dx}\left[\prod_{i=1}^{3} \ell_i(x)\right]\frac{d}{dx}\left[\prod_{i=4}^{7} \ell_i(x)\right]\right] + \frac{d^0}{dx^0}\prod_{i=4}^{7} \ell_i(x)\frac{d^2}{dx^2}\left[\prod_{i=1}^{3} \ell_i(x)\right]$$

The 3^{rd} derivative can be obtained in the same vein such that:

$$\frac{d^3y}{dx^3} = \prod_{i=1}^{3} \ell_i(x)\frac{d^3}{dx^3}\left[\prod_{i=4}^{7} \ell_i(x)\right] + 2\left[\frac{d}{dx}\left[\prod_{i=1}^{3} \ell_i(x)\right]\frac{d^2}{dx^2}\left[\prod_{i=4}^{7} \ell_i(x)\right] + \frac{d^2}{dx^2}\left[\prod_{i=1}^{3} \ell_i(x)\right]\frac{d}{dx}\left[\prod_{i=4}^{7} \ell_i(x)\right]\right]$$

$$+\prod_{i=4}^{7}\ell_i(x)\frac{d^3}{dx^3}\left[\prod_{i=1}^{3}\ell_i(x)\right]+\frac{d}{dx}\left[\prod_{i=4}^{7}\ell_i(x)\right]\frac{d^2}{dx^2}\left[\prod_{i=1}^{3}\ell_i(x)\right]+\frac{d}{dx}\left[\prod_{i=1}^{3}\ell_i(x)\right]\frac{d^2}{dx^2}\left[\prod_{i=4}^{7}\ell_i(x)\right] \tag{14}$$

From the above, the third derivative is obtained as:

$$\frac{d^3y}{dx^3}=\frac{d^0}{dx^0}\left[\prod_{i=1}^{3}\ell_i(x)\right]\frac{d^3}{dx^3}\left[\prod_{i=4}^{7}\ell_i(x)\right]+3\frac{d}{dx}\left[\prod_{i=1}^{3}\ell_i(x)\right]\frac{d^2}{dx^2}\left[\prod_{i=4}^{7}\ell_i(x)\right]$$

$$+3\frac{d^2}{dx^2}\left[\prod_{i=1}^{3}\ell_i(x)\right]\frac{d}{dx}\left[\prod_{i=4}^{7}\ell_i(x)\right]+\frac{d^0}{dx^0}\left[\prod_{i=4}^{7}\ell_i(x)\right]\frac{d^3}{dx^3}\left[\prod_{i=1}^{3}\ell_i(x)\right] \tag{15}$$

This trend can be continued for any other derivatives. At this point we shall substitute results that were obtained in [2, 3] by using the combination of product rule and Leibnitz's theorem, for functions of the forms (10) and (11) such that the first derivative of

$$\left[\prod_{i=1}^{3}\ell_i(x)\right]=\left[\frac{d^0\ell_3}{dx^0}\right]\left(\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right)+\frac{d\ell_3}{dx}\left[\frac{d^0}{dx^0}\prod_{m=1}^{2}\ell_m(x)\right] \tag{16}$$

and the first derivative of

$$\left[\prod_{i=4}^{7}\ell_i(x)\right]= \tag{17}$$

$$\frac{d^0}{dx^0}\prod_{i=4}^{5}\ell_i(x)\left\{\sum_{m=0}^{n=1}\mathtt{C}\,{}^{1}_{m}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^m\ell_7}{dx^m}\right)\right\}+\frac{d^0}{dx^0}\prod_{i=6}^{7}\ell_i(x)\left\{\sum_{m=0}^{n=1}\mathtt{C}\,{}^{1}_{m}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^m\ell_5}{dx^m}\right)\right\}$$

By using equations (16) and (17) in equation (9), we obtain the first derivatives of equation (1) as:

$$\left[\prod_{i=4}^{5}\ell_i(x)\left\{\sum_{m=0}^{n=1}\mathtt{C}\,{}^{1}_{m}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^m\ell_7}{dx^m}\right)\right\}+\prod_{i=6}^{7}\ell_i(x)\left\{\sum_{m=0}^{n=1}\mathtt{C}\,{}^{1}_{m}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^m\ell_5}{dx^m}\right)\right\}\right]\prod_{i=1}^{3}\ell_i(x)$$

$$+\prod_{i=4}^{7}\ell_i(x)\left[\ell_3\left(\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right)+\frac{d\ell_3}{dx}\left[\prod_{m=1}^{2}\ell_m(x)\right]\right] \tag{18}$$

or

$$\left[\frac{d^0}{dx^0}\prod_{i=4}^{5}\ell_i(x)\left\{\sum_{m=0}^{n=1}\mathtt{C}\,{}^{1}_{m}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^m\ell_7}{dx^m}\right)\right\}\right.$$

$$+\frac{d^0}{dx^0}\prod_{i=6}^{7}\ell_i(x)\left\{\sum_{m=0}^{n=1} C\ {}^{1}_{m}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^m\ell_5}{dx^m}\right)\right\}\frac{d^0}{dx^0}\prod_{i=1}^{3}\ell_i(x)$$

(19) $$\left[\frac{d^0}{dx^0}\ell_3\right]\left(\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right)+\frac{d\ell_3}{dx}\left[\frac{d^0}{dx^0}\prod_{m=1}^{2}\ell_m(x)\right]$$

The second derivative can be obtained by substituting for

$$\frac{d^2}{dx^2}\left[\prod_{i=4}^{7}\ell_i(x)\right]\text{ and }\frac{d^2}{dx^2}\left[\prod_{i=1}^{3}\ell_i(x)\right]$$

into equation (13) using the results contained in [2, 3]:

$$\frac{d^2}{dx^2}\left[\prod_{i=4}^{7}\ell_i(x)\right]=\prod_{i=4}^{5}\ell_i(x)\left\{\sum_{m=0}^{n=2} C\ {}^{n}_{m}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^m\ell_7}{dx^m}\right)\right\}$$

$$+2\left\{\sum_{m=0}^{n=1} C\ {}^{n}_{m}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^m\ell_5}{dx^m}\right)\right\}\left\{\sum_{m=0}^{n=1} C\ {}^{n}_{m}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^m\ell_7}{dx^m}\right)\right\}$$

$$+\prod_{i=6}^{7}\ell_i(x)\left\{\sum_{m=0}^{n=2} C\ {}^{n}_{m}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^m\ell_5}{dx^m}\right)\right\}$$

and

(21) $$\frac{d^2}{dx^2}\left[\prod_{i=1}^{3}\ell_i(x)\right]=\prod_{i=1}^{3}\ell_i(x)\left(\sum_{i=0}^{n=2}C_i^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^i\ell_5}{dx^i}\right)\right)$$

$$+2\left\{\prod_{i=1}^{2}\ \ell_1\frac{d\ell_3}{dx}\left[\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^i\ell_5}{dx^i}\right)\right]\right.$$

$$+\left(\ell_3\left[\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^i\ell_5}{dx^i}\right)\right]\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right)$$

$$+\prod_{m=4}^{5}\ell_m(x)\left[\frac{d^2\ell_3}{dx^2}\prod_{m=1}^{2}\ell_m(x)+2\left[\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right]\frac{d\ell_3}{dx}+\ \ell_3\left\{\sum_{i=0}^{n=2}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right\}\right]$$

Equation (21) can be written as:

$$\frac{d^2}{dx^2}\left[\prod_{i=1}^{3}\ell_i(x)\right]=\frac{d^0}{dx^0}\left[\prod_{i=1}^{3}\ell_i(x)\right]\left\{\sum_{i=0}^{n=2}C_i^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^i\ell_5}{dx^i}\right)\right\}+2\left\{\frac{d^0}{dx^0}\left[\prod_{i=1}^{2}\ell_i(x)\right]\right. \quad (22)$$

$$\left[\frac{d\ell_3}{dx}\right]\left[\sum_{i=0}^{n=1} C_i^n \left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^i\ell_5}{dx^i}\right)\right] + \frac{d^0\ell_3}{dx^0}\left[\sum_{i=0}^{n=1} C_i^n \left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^i\ell_5}{dx^i}\right)\right]\left[\sum_{i=0}^{n=1} C_i^n \left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right]$$

$$+\frac{d^0}{dx^0}\left[\prod_{i=4}^{5}\ell_i(x)\right]\left[\frac{d^0}{dx^0}\left[\prod_{i=1}^{2}\ell_i(x)\right]\left[\frac{d^2\ell_3}{dx^2}\right] + 2\left[\sum_{i=0}^{n=1} C_i^n \left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right]\frac{d\ell_3}{dx}\right.$$

$$\left. + \frac{d^0\ell_3}{dx^0}\left\{\sum_{i=0}^{n=2} C_i^n \left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right\}\right.$$

hence, the left hand side of (13) can be obtained as:

$$\prod_{i=1}^{3}\ell_i(x)\left[\prod_{i=4}^{5}\ell_i(x)\left\{\sum_{m=0}^{n=2} \mathtt{C}\begin{matrix} n \\ m\end{matrix}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^m\ell_7}{dx^m}\right)\right\} + 2\left\{\sum_{m=0}^{n=1} \mathtt{C}\begin{matrix} n \\ m\end{matrix}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^m\ell_5}{dx^m}\right)\right\}\right. \tag{23}$$

$$\left\{\sum_{m=0}^{n=1} \mathtt{C}\begin{matrix} n \\ m\end{matrix}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^m\ell_7}{dx^m}\right)\right\} + \prod_{i=6}^{7}\ell_i(x)\left\{\sum_{m=0}^{n=2} \mathtt{C}\begin{matrix} n \\ m\end{matrix}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^m\ell_5}{dx^m}\right)\right\}$$

$$+2\left[\left[\ell_3\left(\sum_{i=0}^{n=1} C_i^n \left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right) + \frac{d\ell_3}{dx}\prod_{m=1}^{2}\ell_m(x)\right]\prod_{i=4}^{5}\ell_i(x)\left\{\sum_{m=0}^{n=1} \mathtt{C}\begin{matrix} 1 \\ m\end{matrix}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^m\ell_7}{dx^m}\right)\right\}\right.$$

$$\left. + \prod_{i=6}^{7}\ell_i(x)\left\{\sum_{m=0}^{n=1} \mathtt{C}\begin{matrix} 1 \\ m\end{matrix}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^m\ell_5}{dx^m}\right)\right\}\right.$$

$$+\prod_{i=4}^{7}\ell_i(x)\left[\prod_{i=1}^{3}\ell_i(x)\left(\sum_{i=0}^{n=2} C_i^n \left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^i\ell_5}{dx^i}\right)\right) + 2\left\{\prod_{i=1}^{2}\ \ell_1\frac{d\ell_3}{dx}\left[\sum_{i=0}^{n=1} C_i^n \left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^i\ell_5}{dx^i}\right)\right]\right.\right.$$

$$+\left(\ell_3\left[\sum_{i=0}^{n=1} C_i^n \left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^i\ell_5}{dx^i}\right)\right]\sum_{i=0}^{n=1} C_i^n \left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right) + \prod_{m=4}^{5}\ell_m(x)\left[\frac{d^2\ell_3}{dx^2}\prod_{m=1}^{2}\ell_m(x)\right.$$

$$\left. +2\left[\sum_{i=0}^{n=1} C_i^n \left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right]\frac{d\ell_3}{dx} + \ \ell_3\left\{\sum_{i=0}^{n=2} C_i^n \left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right\}\right.$$

Thus, (23) gives the second derivative for (1), which is the combination of product rule and Leibnitz's theorem.
Working on the third derivative and making use of some result in [2, 3]

$$\frac{d^3}{dx^3}\left[\prod_{i=1}^{3}\ell_i(x)\right] = \prod_{i=1}^{3}\ell_i\left\{\sum_{i=0}^{n=3}C_i^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^i\ell_5}{dx^i}\right)\right\} \tag{24}$$

$$+3\left\{\sum_{i=0}^{n=2}C_i^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^i\ell_5}{dx^i}\right)\right\}\left[\prod_{i=1}^{2}\ell_i\frac{d\ell_3}{dx}+\ell_3\left\{\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right\}\right]+3\left[\prod_{i=1}^{2}\ell_i\frac{d^2\ell_3}{dx^2}\right.$$

$$\left.+2\frac{d\ell_3}{dx}\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)+\ell_3\left[\sum_{i=0}^{n=2}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right]\right]\left[\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^i\ell_5}{dx^i}\right)\right]$$

$$+\prod_{i=4}^{5}\ell_i\left[\ell_3\left[\sum_{i=0}^{n=3}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right]\right.$$

$$\left.+3\frac{d\ell_3}{dx}\left[\sum_{i=0}^{n=2}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right]+3\frac{d^2\ell_3}{dx^2}\left[\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right]+\prod_{i=1}^{2}\ell_i\frac{d^3\ell_3}{dx^3}\right]$$

and

(25)

$$\frac{d^3}{dx^3}\left[\prod_{i=4}^{7}\ell_i(x)\right]=\prod_{i=4}^{5}\ell_i(x)\left\{\sum_{m=0}^{n=3}\mathtt{C}\,{}^{n}_{m}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^m\ell_7}{dx^m}\right)\right\}+3\left\{\sum_{m=0}^{n=2}\mathtt{C}\,{}^{n}_{m}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^m\ell_5}{dx^m}\right)\right\}$$

$$\left\{\sum_{m=0}^{n=1}\mathtt{C}\,{}^{n}_{m}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^m\ell_7}{dx^m}\right)\right\}+3\left\{\sum_{m=0}^{n=1}\mathtt{C}\,{}^{n}_{m}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^m\ell_5}{dx^m}\right)\right\}$$

$$\left\{\sum_{m=0}^{n=2}\mathtt{C}\,{}^{n}_{m}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^m\ell_7}{dx^m}\right)\right\}+\prod_{i=6}^{7}\ell_i(x)\left\{\sum_{m=0}^{n=3}\mathtt{C}\,{}^{n}_{m}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^m\ell_5}{dx^m}\right)\right\}$$

Thus, by the reason of equations (15), (24) and (25), the third derivative is given as:

$$=\prod_{i=1}^{3}\ell_i(x)\left[\prod_{i=4}^{5}\ell_i(x)\left\{\sum_{m=0}^{n=3}\mathtt{C}\,{}^{n}_{m}\left(\frac{d^{n-m}}{dx^{n-m}}\ell_6\right)\left(\frac{d^m}{dx^m}\ell_7\right)\right\}+3\left\{\sum_{m=0}^{n=2}\mathtt{C}\,{}^{n}_{m}\left(\frac{d^{n-m}}{dx^{n-m}}\ell_4\right)\left(\frac{d^m}{dx^m}\ell_5\right)\right\}\right. \tag{26}$$

$$\left\{\sum_{m=0}^{n=1}\mathtt{C}\,{}^{n}_{m}\left(\frac{d^{n-m}}{dx^{n-m}}\ell_6\right)\left(\frac{d^m}{dx^m}\ell_7\right)\right\}+3\left\{\sum_{m=0}^{n=1}\mathtt{C}\,{}^{n}_{m}\left(\frac{d^{n-m}}{dx^{n-m}}\ell_4\right)\left(\frac{d^m}{dx^m}\ell_5\right)\right\}$$

$$\Bigg\{\sum_{m=0}^{n=2}\mathtt{C}\begin{matrix}n\\m\end{matrix}\left(\frac{d^{n-m}}{dx^{n-m}}\ell_6\right)\left(\frac{d^{m}}{dx^{m}}\ell_7\right)\Bigg\}+\prod_{i=6}^{7}\ell_i\left(x\right)\Bigg\{\sum_{m=0}^{n=3}\mathtt{C}\begin{matrix}n\\m\end{matrix}\left(\frac{d^{n-m}}{dx^{n-m}}\ell_4\right)\left(\frac{d^{m}}{dx^{m}}\ell_5\right)\Bigg\}$$

$$+3\Bigg[\ell_3\Bigg(\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_2}{dx^{i}}\right)\Bigg)+\frac{d\ell_3}{dx}\Bigg[\prod_{m=1}^{2}\ell_m\left(x\right)\Bigg]\Bigg]\Bigg[\prod_{i=4}^{5}\ell_i\left(x\right)\Bigg\{\sum_{m=0}^{n=2}\mathtt{C}\begin{matrix}n\\m\end{matrix}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^{m}\ell_7}{dx^{m}}\right)\Bigg\}$$

$$+2\Bigg\{\sum_{m=0}^{n=1}\mathtt{C}\begin{matrix}n\\m\end{matrix}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^{m}\ell_5}{dx^{m}}\right)\Bigg\}\Bigg\{\sum_{m=0}^{n=1}\mathtt{C}\begin{matrix}n\\m\end{matrix}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^{m}\ell_7}{dx^{m}}\right)\Bigg\}$$

$$+\prod_{i=6}^{7}\ell_i\left(x\right)\Bigg\{\sum_{m=0}^{n=2}\mathtt{C}\begin{matrix}n\\m\end{matrix}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^{m}\ell_5}{dx^{m}}\right)\Bigg\}$$

$$+3\Bigg[\prod_{i=1}^{3}\ell_i\Bigg\{\sum_{i=0}^{n=3}C_i^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_5}{dx^{i}}\right)\Bigg\}+3\Bigg\{\sum_{i=0}^{n=2}C_i^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_5}{dx^{i}}\right)\Bigg\}$$

$$\Bigg[\prod_{i=1}^{2}\ell_i\frac{d\ell_3}{dx}+\ell_3\Bigg\{\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_2}{dx^{i}}\right)\Bigg\}\Bigg]+3\Bigg[\prod_{i=1}^{2}\ell_i\frac{d^2\ell_3}{dx^2}+2\frac{d\ell_3}{dx}\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_2}{dx^{i}}\right)$$

$$+\ell_3\Bigg[\sum_{i=0}^{n=2}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_2}{dx^{i}}\right)\Bigg]\Bigg[\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_5}{dx^{i}}\right)\Bigg]+\prod_{i=4}^{5}\ell_i\Bigg[\ell_3\Bigg[\sum_{i=0}^{n=3}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_2}{dx^{i}}\right)\Bigg]$$

$$+3\frac{d\ell_3}{dx}\Bigg[\sum_{i=0}^{n=2}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_2}{dx^{i}}\right)\Bigg]+3\frac{d^2\ell_3}{dx^2}\Bigg[\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_2}{dx^{i}}\right)\Bigg]+\prod_{i=1}^{2}\ell_i\frac{d^3\ell_3}{dx^3}$$

$$\Bigg[\prod_{i=4}^{5}\ell_i\left(x\right)\Bigg\{\sum_{m=0}^{n=1}\mathtt{C}\begin{matrix}1\\m\end{matrix}\left(\frac{d^{n-m}\ell_6}{dx^{n-m}}\right)\left(\frac{d^{m}\ell_7}{dx^{m}}\right)\Bigg\}+\prod_{i=6}^{7}\ell_i\left(x\right)\Bigg\{\sum_{m=0}^{n=1}\mathtt{C}\begin{matrix}1\\m\end{matrix}\left(\frac{d^{n-m}\ell_4}{dx^{n-m}}\right)\left(\frac{d^{m}\ell_5}{dx^{m}}\right)\Bigg\}\Bigg]$$

$$+\prod_{i=4}^{7}\ell_i\left(x\right)\Bigg[\prod_{i=1}^{3}\ell_i\Bigg\{\sum_{i=0}^{n=3}C_i^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_5}{dx^{i}}\right)\Bigg\}+3\Bigg\{\sum_{i=0}^{n=2}C_i^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_5}{dx^{i}}\right)\Bigg\}$$

$$\Bigg[\prod_{i=1}^{2}\ell_i\frac{d\ell_3}{dx}+\ell_3\Bigg\{\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_2}{dx^{i}}\right)\Bigg\}\Bigg]+3\Bigg[\prod_{i=1}^{2}\ell_i\frac{d^2\ell_3}{dx^2}+2\frac{d\ell_3}{dx}\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_2}{dx^{i}}\right)$$

$$+\ell_3\Bigg[\sum_{i=0}^{n=2}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_2}{dx^{i}}\right)\Bigg]\Bigg[\sum_{i=0}^{n=1}C_i^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_5}{dx^{i}}\right)\Bigg]+\prod_{i=4}^{5}\ell_i\Bigg[\ell_3\Bigg[\sum_{i=0}^{n=3}C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^{i}\ell_2}{dx^{i}}\right)\Bigg]$$

$$+3\frac{d\ell_3}{dx}\left[\sum_{i=0}^{n=2} C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right]+3\frac{d^2\ell_3}{dx^2}\left[\sum_{i=0}^{n=1} C_i^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\right]+\prod_{i=1}^{2}\ell_i\frac{d^3\ell_3}{dx^3}$$

Thus, we can generate this for the $(k+1)^{th}$ order derivative for the equation in (1) by using Mathematical Induction.

The following theorem is formulated from the above assertion:

Theorem 2.1:

Let $\mathcal{Y}$ be defined as in (1), (2), (3) and (4). Then, the n^{th} order derivative of $\mathcal{Y}$ is given as:

$$\frac{d^n y}{dx^n}=\sum_{r=0}^{n}\mathtt{C}_r^n\varphi^r\phi^{n-r} \tag{27}$$

$$where \quad \varphi=\frac{d}{dx}\left[\prod_{i=1}^{3}\ell_i(x)\right] \quad and \quad \phi=\frac{d}{dx}\left[\prod_{i=4}^{7}\ell_i(x)\right]$$

for which the n^{th} order derivatives φ *and* ϕ *are given as*;

$$\frac{d^n}{dx^n}\left[\prod_{i=1}^{3}\ell_i(x)\right]=\sum_{r=0}^{n+1}\mathtt{C}_r^{n+1}\left[\sum_{i=0}^{n}\mathtt{C}_r^n\left(\frac{d^{n-i}\ell_1}{dx^{n-i}}\right)\left(\frac{d^i\ell_2}{dx^i}\right)\frac{d^r\ell_3}{dx^r}\right] \tag{28}$$

and

(29)
$$\frac{d^n}{dx^n}\left[\prod_{i=4}^{7}\ell_i(x)\right]=\sum_{r=0}^{n+1}\mathtt{C}_r^{n+1}\sum_{r=0}^{n+1}\mathtt{C}_r^{n+1}\left(\frac{d^{n-i}\ell_6}{dx^{n-i}}\right)\left(\frac{d^i\ell_7}{dx^i}\right)\left[\sum_{i=0}^{n}\mathtt{C}_r^n\left(\frac{d^{n-i}\ell_4}{dx^{n-i}}\right)\left(\frac{d^i\ell_5}{dx^i}\right)\right]$$

CONCLUSION

A computer friendly algorithm may be generated and be used to develop software, with an interface which is capable of computing the n^{th} order derivatives of the class of function presented in this work. This approach does not require the knowledge of the previous derivatives of the function in question before given the next higher order derivatives.

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