Nigerian Journal of Mathematics and Applications
Volume **27**, (2018), 108 − 115. *Printed by Unilorin press*
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Unsteady Mhd Oscillatory Flow Through A Porous Channel Saturated With Porous Medium

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Abstract

In this paper, we investigates the effect of suction/injection on the unsteady oscillatory flow through a vertical channel with non-uniform wall temperature. The fluid is subjected to a transverse magnetic field and the velocity slip at the lower plate is taken into consideration. Exact solutions of the dimensionless equations governing the fluid flow are obtained and the effects of the flow parameters on temperature, velocity profiles, skin friction and rate of heat transfer are discussed and shown graphically. It is interesting to note that skin friction increases on both channel plates as injection increases on the heated plate.

1. Introduction

The study of oscillatory flow of an electrically conducting fluid through a porous channel filled with saturated porous medium is important in many physiological flows and engineering applications. The study finds its application in magneto-hydrodynamics (MHD) generators, arterial blood flow, petroleum engineering and many more.

Several authors have studied the flow and heat transfer in oscillatory fluid problems. To mention just a few, Makinde and Mhone [9] investigated the forced convective MHD oscillatory fluid flow through a channel filled with porous medium,

Received: 30/06/2018, Accepted: 24/09/2018, Revised: 29/11/2018.
2015 *Mathematics Subject Classification.* 76S06, & 36Q60.
Key words and phrases. Oscillatory flow, Porous medium, Magnetic field, Fluid slip, Suction/injection
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analysis were based on the assumption that the plays are impervious. In a related study, Mehmood and Ali [10] investigated the effect of slip on the free convective oscillatory flow through vertical channel with periodic temperature and dissipative heat. In addition, Chauchan and Kumar [4] studied the steady flow and heat transfer in a composite vertical channel. In [11], Palani and Abbas investigated the combined effects of magneto-hydrodynamics and radiation effect on free convection flow past an impulsively started isothermal vertical plate using the Rosseland approximation. Hussain *et al.* [5] presented analytical study of oscillatory second grade fluid flow in the presence of a transverse magnetic field and many more.

In all the studies above, the channel walls are assumed to be impervious. This assumption is not valid in studying the blood flow in miniature level where digested food parties are diffused into the blood stream through the wall of the blood capillary. Hence, due to several other important suction/injection controlled applications. There have been several studies on the convective heat transfer through porous channel, for instance, Avathi *et al.* [3] investigated the unsteady flow of viscous fluid a horizontal composite channel whose half width is filled with porous medium. Jha and Ajibade [6] presented the effects of suction and injection on hydrodynamics of oscillatory fluid through parallel plates. The same authors extended the problem to heat generating/absorbing fluids in [7] while in [8] the effect of vscous dissipation of the free convective flow with time dependent boundary condition was investigated. More recently, Adesanya and Makinde [2] investigated the effect of radiative heat transfer on the pulsatile couple stress fluid flow with time dependent boundary condition on the heated plate. It is well known that the no slip condition is not realistic in some flows involving Nano-channel, micro-channel and flows over coated plates with hydrophobic substances. In view of this, Adesanya and Gbadeyan [1] studied the flow and heat transfer of steady non-Newtonian fluid flow noting the fluid slip in the porous channel.

After careful survey of the literature, it was observed that the effect on suction/injection on the slip flow of oscillatory hydromagnetic fluid through a channel filled with saturated porous medium received little attention. Therefore, the specific objective of this paper is to extend the work done in Mehood and Ali [10], to include the effect of suction/injection at the cold plate. The rest of the paper is organized as follows: section 2 provides adequate information on the formulation and non-dimensionalization of the problem. In section 3, the method of solution problem is presented. Section 4 presents the results and discussion while section 5 concludes the work.

2. Mathematical Analysis

Consider the unsteady laminar flow of an incompressible viscous electrically conducting fluid through a channel with slip at the cold plate. The fluid is subjected to an external magnetic field applied across the normal to the channel. It is assumed that the fluid has small electrical conductivity and the electro-magnetic force produced is also very small. The flow is subjected to suction at the cold wall and injection at the heated wall. we choose a Cartesian coordinate system $(x^{'}, y^{'})$ where $x^{'}$ lies along the center of the channel, $y^{'}$ is the distance measured in the normal section such that $y^{'} = a$ is the channel's half width. Under the usual business approximation the equations governing the flow are:

$$\frac{\partial u^{'}}{\partial t^{'}} - v_0 \frac{\partial u^{'}}{\partial y^{'}} = -\frac{1}{\rho}\frac{dP^{'}}{\partial x^{'}} + v\frac{\partial^2 u^{'}}{\partial y^{'2}} - \frac{v}{K}u^{'} - \frac{\sigma_e B_0^2}{\rho}u^{'} + g\beta(T - T_0) \tag{1}$$

$$\frac{\partial T^{'}}{\partial t^{'}} - v_0 \frac{\partial T^{'}}{\partial y^{'}} = -\frac{k_f}{\rho C_p}\frac{\partial^2 T^{'}}{\partial y^{'2}} + \frac{4\alpha^2}{\rho C_p}(T - T_0) \tag{2}$$

with the boundary conditions:

$$u^{'} = \frac{\sqrt{K}}{\alpha_s}\frac{\partial u^{'}}{\partial t^{'}}, T = T_0 \ on \ y^{'} = 0 \tag{3}$$

and

$$u^{'} = 0, T^{'} = T_0 \ on \ y = a \tag{4}$$

where $t^{'}$ denotes time, $u^{'}$ axial velocity, v_0 constant horizontal velocity, ρ fluid density, $P^{'}$ fluid pressure, v kinematic viscosity, K porous permeability, σ_e electrical conductivity, B_0 magnetic field intensity, g gravitational acceleration, β volumetric expansion, C_p specific heat at constant pressure, α term due to thermal radiation, k thermal conductivity, $T^{'}$ fluid temperature and T_0 referenced fluid temperature.

Introducing the dimensionless parameters and variables given below:

$$(x, y) = \frac{(x^{'}, y^{'})}{h}, u = \frac{hu^{'}}{v}, t = \frac{vt^{'}}{h^2}, p = \frac{h^2 p^{'}}{pv^2}, Gr = \frac{g\beta(T_1 - T_0)h^3}{v^2}, pr = \frac{pC_p v}{h},$$

$$\theta = \frac{T - T_0}{T - T_0}, \delta = \frac{4\alpha^2 h^2}{pC_p v}, \gamma = \frac{\sqrt{k}}{\alpha_s h}, Ha^2 = \frac{\sigma_e B_0^2 h^2}{pv}, Da = \frac{k}{h^2}, s = \frac{v_0 h}{v}$$

we obtain the dimensionless equations:

(5) $$\frac{\partial u}{\partial t} - s\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\partial^2 u}{\partial y^2} - \left(Ha^2 + \frac{1}{Da}\right)u + Gr\theta$$

(6) $$\frac{\partial \theta}{\partial t} - s\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + \delta\theta$$

with the appropriate boundary conditions:

(7) $$u = \gamma\frac{du}{dy}, \quad \theta = 0 on y = 0$$

(8) $$u = 0, \quad \theta = 0 on y = 1$$

where Da is the Darcy parameter, s is the suction/injection parameter, Ha^2 is Hartmann's number, Gr is the Grashof number, Pr is the Prandtl number, δ is the thermal radiation parameter and γ is the Navier slip parameter.

3. Method of Solution

The oscillatory nature of the fluid flow suggest the solutions of (5)-(8) in the form:

(9) $$-\frac{dP}{dx} = \lambda e^{i\omega t}, u(t,y) = u_0(y)e^{i\omega t}, \theta(t,y) = \theta_0(y)e^{i\omega t}$$

where λ is any positive constant, ω is the frequency of oscillation. Using (9) in (5)-(8) we obtain:

(10) $$u_0'' + su_0' - \left(Ha^2 + \frac{1}{Da} + i\omega\right)u_0 = -\lambda - Gr\theta_0; \quad u_0(0) = \gamma u_0'(0), u(1) = 0$$

(11) $$\theta_0'' + sPr\theta_0' + (\delta - i\omega)Pr\theta_0 = 0; \quad \theta_0(0) = 0, \theta_0(1) = 1$$

solving (11) gives:

(12) $$\theta(t,y) = (A_0 e^{m_1 y} + B_0 e^{m_2 y})e^{i\omega t}$$

The rate of heat transfer is given by:

(13) $$Nu = \frac{\partial \theta}{\partial y} = (A_0 m_1 e^{m_i y} + B_0 m_2 e^{m_2 y})e^{i\omega t}$$

Also, the exact solution of (11) is

(14) $$u(t,y) = \{A_1 e^{m_3 y} + B_1 e^{m_4 y} + Q_0 + Q_1 e^{m_1 y} + Q_2 e^{i\omega t}\}e^{i\omega t}$$

The shear stress is given by the relation

$$S = \frac{\partial u}{\partial y} = (A_1 m_3 e^{m_3 y} + B_1 m_4 e^{m_4 y} + m_1 Q_1 e^{m_1 y} + m_2 Q_2 e^{m_2 y}\} e^{i\omega t} \tag{15}$$

all the constants are defined in the appendix.

4. Results and Discussion

In this paper, the study of oscillatory hydromagnetic fluid flow through a permeable channel filled with a porous medium is studied. The flow is due to free convection and increasing pressure gradient through a vertical channel. The effect of the suction/injection parameter on the temperature of the fluid within the channel is shown in fig. 1. from the plot, it is observed that fluid temperature is linearly distributed within the channel in the absence of suction/injection. However, as injection increases on the heated plate, fluid temperature increases within the channel and the linearity observed at s=0 has given way to concave distribution. The concavity is as result of the direction of heat flow from the heated plate towards the cold plate.

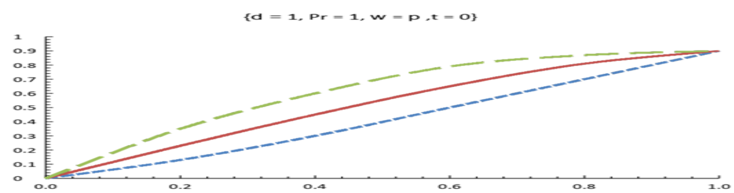

Figure 1: Effect of suction/injection parameter on fluid temperature

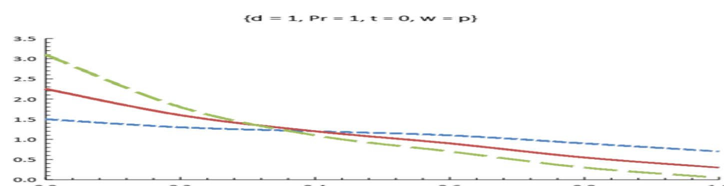

Figure 2: Effect of suction/injection parameter on the rate of heat transfer

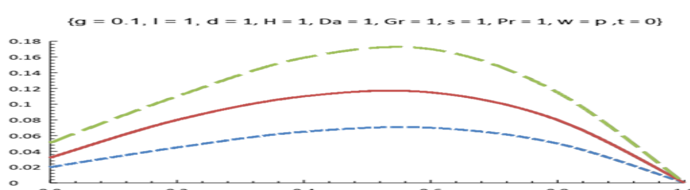

Figure 3: Effect of oscillations on fluid velocity

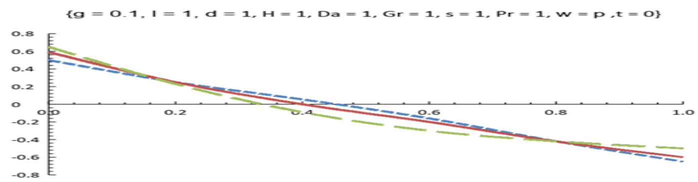

Figure 4: Effect of suction/injection on the skin-fraction across the channel

Figure 1 shows that the fluid absorbs its own radiations therefore leading to increase in the fluid temperature, the influence of suction/injection is what accounts for the nonlinearity of the temperature distribution in the channel. Also, as observed Figure 2 shows the plot of heat transfer within the channel as observed in the heated channel the rate of heat transfer from the heated channel decreases and increases along the cold wall. this is physically true since rate of heat transfer from the heated plate to the fluid and the heat is transferred to the cold plate. Therefore, the fluid absorbs its own emission which is evident in rise in fluid temperature. The plot of variations in the fluid slip parameter shows that increase in slip enhances the flow at the surface of the cold wall. A general increase in fluid velocity is seen as the oscillation parameter increases in Figure 3. while an increase in the thermal radiation parameter increases the fluid velocity due to convection currents that the strengthened with increase in temperature that goes with thermal radiation absorption as observed in Figure 4 and it shows that an increase in the suction/injection parameter increases the skin friction at the both walls.

Conclusion

In this paper, the oscillatory flow of hydromagnetic fluid through a porous channel filled with porous medium is studies in the optically thin thermal radiation limit. The obtained result reduced to that presented in [10] when $s = 0$. Additionally, we conclude that an increase in the suction/injection decreases that heat transfer at the heated plate and increase it at the cold wall while the skin friction increases on both plates as s increases.

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Appendix

$$m_1 = \frac{-sPr + \sqrt{s(Pr)^2 - 4Pr(\delta - i\omega)}}{2}$$

$$m_2 = \frac{-sPr - \sqrt{s(Pr)^2 - 4Pr(\delta - i\omega)}}{2}$$

$$m_3 = \frac{-s + \sqrt{s^2 + 4(H^2 + 1/Da + i\omega)}}{2}$$

$$m_4 = \frac{-s - \sqrt{s^2 + 4(H^2 + 1/Da + i\omega)}}{2}$$

$$A_0 = -\frac{1}{e^{m2} - e^{m1}}$$

$$B_0 = \frac{1}{e^{m2} - e^{m1}}$$

$$Q_0 = -\frac{\lambda}{H^2 + 1/Da + i\omega}$$

$$Q_1 = -\frac{\lambda}{m_1^2 + sm_1 - (H^2 + 1/Da + i\omega)}$$

$$Q_2 = -\frac{GrB_0}{m_2^2 + sm_2 - (H^2 + 1/Da + i\omega)}$$

$$n_0 = Q_0 + Q_1 + Q_2$$

$$n_1 = m_1\gamma Q_1 + Q_2 m_2 \gamma$$

$$n_2 = Q_0 + Q_1 e^{m1} + Q_2 e^{m2}$$

$$B_1 = \frac{\left(n_2 + \frac{(n_1 - n_0)e^{m_3}}{1 - m_3\gamma}\right)}{\left(e^{m_4} + \frac{(m_4\gamma - 1)}{1 - m_3\gamma}\right)e^{m_3}}$$

$$A_1 = \frac{B_1(m_4\gamma - 1) + n_1 - n_0}{1 - m_3\gamma}$$