Nigerian Journal of Mathematics and Applications
Volume **26**, (2017), 128 − 137. *Printed by Unilorin press*
©*Nig. J. Math. Appl.* *http://www.njmaman.com*

Investigating the Effect of Damping Coefficients on Euler-Bernoulli Beam Subjected to Partially Distributed Moving Load

M. A. Usman* , B. T. Ogunsan, S. T. Okusaga and O. O. Solanke

Abstract

In this paper, the effect of damping coefficients on Euler-Bernoulli Beam subjected to distributed load is studied. The partial differential equation of order five was transformed to ordinary differential equation of order two using finite fourier sine transform. Finite difference method was used to solve the resulting differential equation of order two. It was found that the amplitude decreases as the speed of the load increases with the introduction of damping coefficient and flexural rigidity while the amplitude of the deflection increases when there is no damping.

1. Introduction

Beams are very important elements in land mechanical and aeronautical engineering Beam is a piece of horizontal structure that is usually supported at both ends. It can be in form of wood, metal or plastic, this is concern with the theory describing the respond of elastic structure under the influence of partially distributed moving loads. The most obvious example of structure subjected to partially distributed moving loads is highway and railways bridges. Furthermore,

Received: 12/01/2017, Accepted: 13/04/2017, Revised: 07/06/2017.
2015 *Mathematics Subject Classification.* 44Axx & 74S05. * Corresponding author.
Key words and phrases. Damping Coefficients; Flexural rigidity; Lateral deflection or Amplitude; Partially distributed load
Department of Mathematical Sciences, Olabisi Onabanjo University, Ago Iwoye, Nigeria
E-mail: usmanma@yahoo.com

there is a form of interaction between the motion of the bridge and suspension of the vehicle. Some load applied statistically especially if the moving riding surface is uneven, [3, 6, 8,9].

In the analysis of elastically supported beams, the elastic support is provided by a load bearing medium referred to as the foundation along the length of the beam. Such conditions of support can be found in large variety of geotechnical problems. There are two basic types characterized by the fact that the pressure in the foundation is proportional at every point to the deflection occurring at that point and is independent of pressure or deflection produced at other point, [5, 15, 17, 19].

In general, there are two types of motion of elastic structure [4, 5]:

(1) the Thick-structure theory which account for the effect of shear deformation and rotatory inertia while;
(2) the classical thin structures neglect the effects of shear information and rotary inertia.

This research work has therefore been motivated by the above stated observations. An investigation into the effect of damping coefficients on Euler-Bernoulli beam resting on a Winkler foundation subjected to partially distributed moving load is presented, the resulting coupled Partial differential equations of model one and model two are solved using finite difference method and series solution method [18, 19].

All loads on a beam act parallel to an axis that is transverse to its longitudinal axis. The length of a beam is much longer than its width and depth. Beam theory provides equations for deflection and internal forces of the beam. Loads are generally forces acting on a structure. When loads act on a structure they produce stress and deformations [6, 23].

The problem of carrying out a dynamic analysis of the reactions of structures under moving loads is known as moving load problem such moving load problems are of practical important. The most obvious example of structures subjected to moving loads is highway and railway bridges. The pertinent analysis is, however, complicated by the fact that the mass of a moving vehicle or locomotive is usually large compare with that of the bridge itself [7, 8, 9, 23].

Moving load problems may be grouped into two main classes. The first class consist of problems involving concentrated forces (or point masses) moving with a specified or an unspecified velocity while the other class deals with the problem of vibration analysis of structures due to partially distributed uniformly moving forces (or masses).One obvious application for the analysis of the second class is the study of vibration of a bridge under a travelling train. Also, since no point or concentrated mass exists physically, consideration of a load distribution interval enhances the reality of problem formulation involving the second class.

Beams are generally known to be of four main types viz [23]:

(1) Euler-Bernoulli beams
(2) Shear beams
(3) Rayleigh beams and
(4) Timoshenko beams.

Euler- Bernoulli beams are the simplest and most common ones. They are known not to possess the effect of both shear deformation and rotatory inertia. Shear beams are beams in which only the effects of shear deformation are retained while Rayleigh beams take into consideration the effect of rotatory inertial only. For Timoshenko beams, the effects of both Shear deformation and rotatory inertia are retained. Beams vibrations described by the Timoshenko model have been studied over the years by many authors [11,12,13,23]. The Timoshenko model is an extension of the Euler-Bernoulli model by taking into account two additional effects; shearing force effect and rotatory motion effect. In any beam except one subjected to pure bending only, a deflection due to the shear stress occurs. The exact solution to the beam vibration problem requires this deflection to be considered [15, 16, 23].

Beams are the most common type of structural components, particularly in civil and mechanical engineering. A beam is a bar-like structural member whose primary function is to support transverse loading and carry it to the supports [7, 8, 9, 23]. The main difference of beams with respect to bars is the increased order of continuity required for the assumed transverse-displacement functions to be admissible [6, 23]. Not only must those functions be continuous but they must possess continuous X first derivatives. To meet this requirement both deflections and slopes are matched at nodal points. Slopes may be viewed as rotational degrees of freedom in the small-displacement assumptions used here [17, 23].

Beams can vary greatly in their geometry and composition [3, 23]. For instance, a beam may be straight or curved. It may be made entirely from the same material (homogenous), or it may be composed of different materials (composite) [16, 23]. Some of these things make analysis difficult but many engineering applications involve cases that are not so complicated. Analysis is simplified if:

The beam is originally straight;

The beam experiences only linear elastic deformation; and

The beam is slender (its length to height ratio is greater than 10)

Only small deflections are considered (maximum deflection is less than 1/10 span).

In engineering, deflection is the degree to which a structural element is displaced under a load [14, 23]. It may refer to an angle or a distance and is inversely proportional to moment of inertia, modulus of elasticity is constant for all structural steels, so the larger the moment of inertia, the smaller the deflection. Beams are traditionally descriptions of building or civil engineering structural elements but smaller structures like trucks or automobile frames, machine frames and other mechanical or structural systems contain beam structures that are designed and analyzed in a similar fashion [21, 22, 23].

2. MATHEMATICAL FORMULATION

Nomenclature

M	mass of the load
g	acceleration due to gravity
ε	fixed length of the load
ϵ	length of the beam
$g(x,t)$	distributed load (force per length)
$K_D I$	damping coefficient
I	Second moment of inertia
$W(x,t)$	Lateral Deflection of beam measured upward from equilibrium position with the load
A	Cross sectional area of the beam
ρ	Mass density of the load
E	Modulus of Elasticity
X	Axial Coordinate
K	The coefficient of Winkler Foundation (force per length square)
m	Constant Mass per unit length of the beam
t	time
EI	flexural rigidity of the beam
AD	Viscoelastic material constant
KD	Kelving-voigt damping coefficient

2.1. **GOVERNING EQUATION.** Consider a Euler-Bernoulli beam carrying Partially distributed load advancing uniformly along the beam with a constant velocity V. It is assumed the load is situated at the centre supported at $t = 0$. The load M is uniformly distributed on a fixed lengthon the beam. Further, the beam has a simple support at both ends.
The behaviour of the Euler-Bernoulli beam carrying the time carrying force is govern by the partial differential equation is given by:

$$EI\frac{\partial^4 W(x,t)}{\partial x^4} + \rho A\frac{\partial^2 W(x,t)}{\partial t^2} = q(x,t) \tag{1}$$

If the above include damping effects which are associated to the EBB, the corresponding PDE is

$$K_D I\frac{\partial^5 W(x,t)}{\partial x^4 \partial t} + EI\frac{\partial^4 W(x,t)}{\partial x^4} + \rho A\frac{\partial^2 W(x,t)}{\partial t^2} + A_D\frac{\partial W(x,t)}{\partial t} = q(x,t) \tag{2}$$

where

$$(3)\qquad q\left(x,t\right)=\frac{1}{\epsilon}\left[-Mg-M\frac{d^{2}\mathrm{W}}{dt^{2}}\right]\left[H\left(x-\varepsilon+\frac{\epsilon}{2}\right)-H\left(x-\varepsilon-\frac{\epsilon}{2}\right)\right]$$

and H is the Heaviside function such that

$$(4)\qquad H\left(x-\varepsilon+\frac{\epsilon}{2}\right)-H\left(x-\varepsilon-\frac{\epsilon}{2}\right)=\begin{cases}0 & x<\varepsilon-\frac{\epsilon}{2}\\ 1 & x>\varepsilon-\frac{\epsilon}{2}\end{cases}$$

$$(5)\qquad H\left(x-\varepsilon+\frac{\epsilon}{2}\right)-H\left(x-\varepsilon-\frac{\epsilon}{2}\right)=\begin{cases}0 & x<\varepsilon+\frac{\epsilon}{2}\\ 1 & x>\varepsilon+\frac{\epsilon}{2}\end{cases}$$

$$(6)\qquad \left[H\left(x-\varepsilon+\frac{\epsilon}{2}\right)-H\left(x-\varepsilon-\frac{\epsilon}{2}\right)\right]f\left(x\right)=\begin{cases}0 & if\ x<\varepsilon-\frac{\epsilon}{2}\\ f\left(x\right) & if\ \varepsilon-\frac{\epsilon}{2}<x<\varepsilon+\frac{\epsilon}{2}\\ 0 & if\ x>\varepsilon+\frac{\epsilon}{2}\end{cases}$$

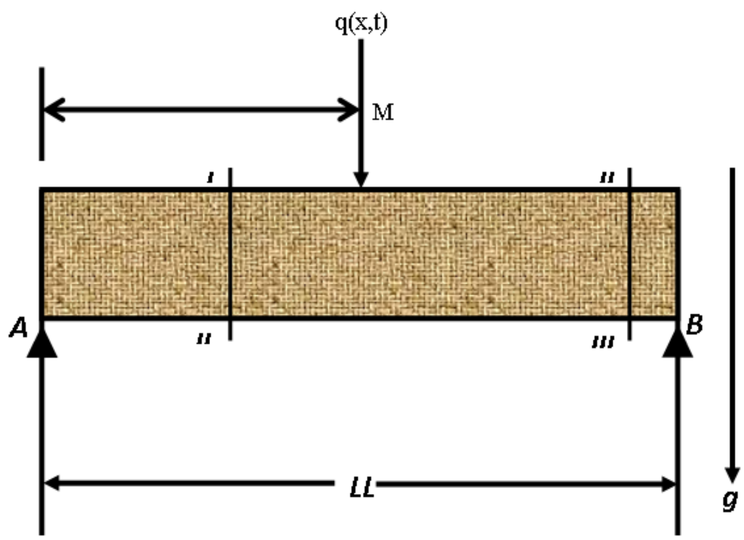

Figure 2.1: The Uniform partially distributed moving load M on the beam AB

The Differential operator $\frac{\mathrm{d}^2\mathrm{W}}{\mathrm{dt}^2}$ is defined as:

$$(7)\qquad \frac{\mathrm{d}^2\mathrm{W}}{\mathrm{dt}^2}=\frac{\partial^2\mathrm{W(x,t)}}{\partial\mathrm{t}^2}+2V\frac{\partial^2\mathrm{W(x,t)}}{\partial\mathrm{x}\,\partial\mathrm{t}}+V^2\frac{\partial^2\mathrm{W(x,t)}}{\partial\mathrm{x}^2}$$

Substituting equation (3) and (4) into equation (2), then the governing equation becomes:

$$\begin{aligned} &K_D I\frac{\partial^5 W(x,t)}{\partial x^5 \partial t} + EI\frac{\partial^4 W(x,t)}{\partial x^4} + \rho A\frac{\partial^2 W(x,t)}{\partial t^2} + A_D\frac{\partial W(x,t)}{\partial t}, \\ &= \frac{1}{\epsilon}\left[-Mg - M\frac{\partial^2 \mathrm{W(x,t)}}{\partial \mathrm{t}^2} + 2MV\frac{\partial^2 \mathrm{W(x,t)}}{\partial \mathrm{x}\,\partial \mathrm{t}} + MV^2\frac{\partial^2 \mathrm{W(x,t)}}{\partial \mathrm{x}^2}\right] \end{aligned} \tag{8}$$

2.2. **BOUNDARY CONDITIONS.** The boundary conditions are given as:

$$\left.\begin{aligned} W(0,\ t) = 0 = W(L,\ t) \\ W_{xx}(0,\ t) = 0 = W_{xx}(L,\ t) \end{aligned}\right\} \tag{9}$$

Without loss of generality, consider the initial condition of the form:

$$W(0,\ t) = 0 = W(L,\ t) \tag{10}$$

2.3. **METHOD OF SOLUTION.** To obtain a solution for the differential equation of equation (8), we used the finite Fourier sine transform to reduce the equation from fifth order partial differential equation to a second order ordinary differential equation. The finite Fourier sine transform is given by:

$$W(m,t) = \int_0^L W(x,t)\sin\frac{m\pi}{l}\ x\ dx \tag{11}$$

with the inverse

$$W(x,t) = \frac{2}{L}\int_{m=1}^{\infty} W(m,t)\sin\frac{m\pi}{l}\ x\ dx \tag{12}$$

Applying equation (6) into equation (8) taking into account the boundary conditions (9), we have

$$\begin{aligned} &K_D I\left(\tfrac{m\pi}{l}\right)^4\tfrac{dW(x,t)}{dt} + EI\left(\tfrac{m\pi}{l}\right)W(x,t) + \mathrm{A}\ \tfrac{d^2W(x,t)}{dt^2} + A_D\tfrac{dW(x,t)}{dt} \\ &= -\tfrac{\mathrm{Mg}}{\epsilon} - \tfrac{\mathrm{M}}{\epsilon}\tfrac{d^2W(x,t)}{dt^2} - \tfrac{\mathrm{2MV}}{\epsilon}\left(\tfrac{m\pi}{l}\right)\tfrac{dW(x,t)}{dt} - \tfrac{\mathrm{MV}^2}{}\left(\tfrac{m\pi}{l}\right)^2 W(x,t) \end{aligned} \tag{13}$$

Equation (13) becomes

$$\frac{d^2W(x,t)}{dt^2} + \mathrm{R}\frac{dW(x,t)}{dt} + \mathrm{S}\ W(x,t) = \frac{-\mathrm{Mg}}{\epsilon} \tag{14}$$

where

$$R = \frac{K_D I\left(\frac{m\pi}{l}\right) + \frac{2MV}{\epsilon} + A_D}{\rho A + \frac{M}{\epsilon}} \tag{15}$$

and

$$S = \frac{EI\left(\frac{m\pi}{l}\right) - \frac{MV^2}{\epsilon} + \left(\frac{m\pi}{l}\right)^2}{\rho A + \frac{M}{\epsilon}} \tag{16}$$

3. NUMERICAL SOLUTION

The reduced second order differential equation (14) is solved by applying finite difference method and equation (14) becomes:

$$\left.\begin{array}{l} \frac{d^2\mathrm{W(x,t)}}{dt^2} = \frac{1}{h^2}\left(w_{j+1} - 2W_j + W_{j-1}\right) \\ \frac{\mathrm{d}w_{j+1}-2W_j+W_{j-1}}{dt} = \frac{1}{2h}\left(W_{j+1-}W_j - 1\right) \end{array}\right\} \tag{17}$$

$$W_{j+1-}W_j - 1 + R\frac{(\mathrm{W_{j+1}-W_{j-1}})}{2h} + S\ W_j = \frac{-M_g}{\epsilon} \tag{18}$$

Multiply (18) by h^2, we have

$$W_{j+1} - 2W_j + W_{j-1} + \frac{Rh}{2}\left(W_{j+1} - W_{j-1}\right) + \ Sh^2W_j = \frac{-Mgh^2}{\epsilon} \tag{19}$$

$$\left(1 + \frac{Rh}{2}\right)W_{j+1} + \left(1 - \frac{Rh}{2}\right)W_{j-1} + \left(Sh^2 - 2\right)W_j = \frac{-Mgh^2}{\epsilon} \tag{20}$$

Multiply equation (18) through by 2, we have

$$\left(2 + Rh\right)W_{j+1} + \left(2 - Rh\right)W_{j-1} + \left(2Sh^2 - 4\right)W_j = \frac{-2Mgh^2}{\epsilon} \tag{21}$$

Rearrange equation (21) we have

$$W_{j+1} = \frac{1}{(2 + Rh)}\left[\frac{-2Mgh^2}{\epsilon} + \left(4 - 2Sh^2\right)W_j + \left(Rh - 2\right)W_{j-1}\right] \tag{22}$$

In view of equation (12), we have

$$W\left(x,t\right) = \frac{2}{L}\sum_{m=1}^{\infty}\left[\frac{1}{(2 + Rh)}\right]\left[\begin{array}{c} \frac{-2Mgh^2}{\epsilon} + \left(4 - Sh^2\right)W_j \\ + \left(Rh - 2\right)W_{j-1} \end{array}\right]\sin\frac{m\pi x}{L} \tag{23}$$

The above system (23) is obtained from the finite difference method and then run by a MATLAB package.

3.1. **Numerical Results and Discussion.** The finite difference system (3.20) is solved numerically. The package was used for the following data: M= 70kg, m= 7.04kg, h = 1.05m, L= 10m, V= 3.3m/s, K_D = 0, 0.2 MN/m^3 and 0.4 NM/m^3, E = 2x10^{-11}, g = 9.8m/s, I = 1.04 x 10^6, π = 3.142.

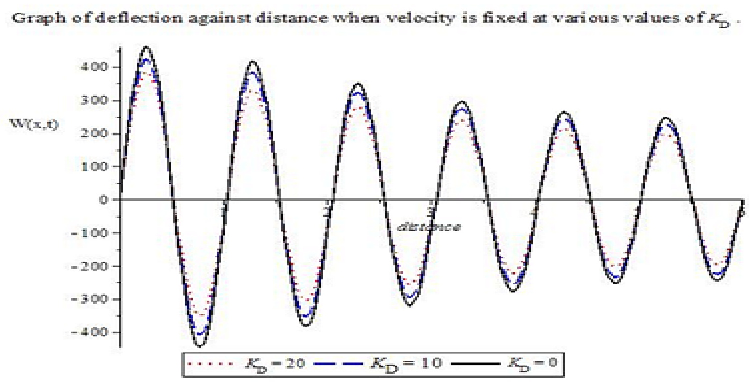

Figure 3.1: Deflection of Beam at various values of K_D when the speed of the moving load V is fixed

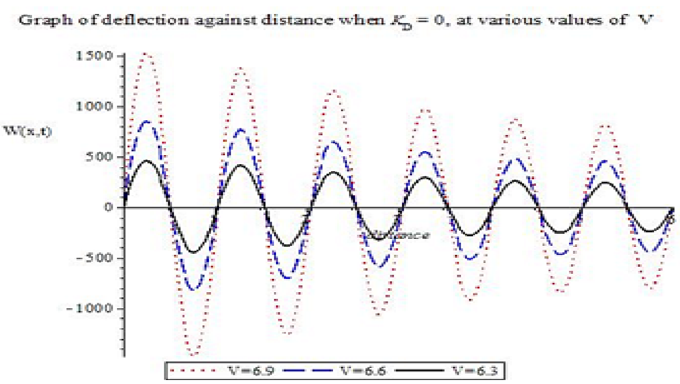

Figure 3.2: Effect of Speed of the Moving load on the Beam when K_D=0

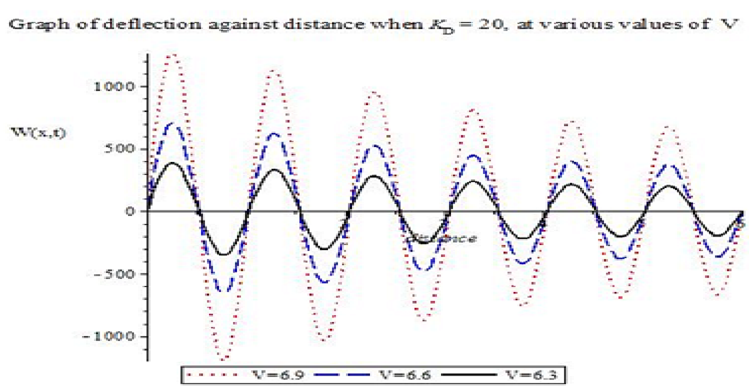

Figure 3.3: Effect of Speed of the Moving load on the Beam when $K_D = 20$

3.2. **Discussion of Results.** In this article, we discuss the effective damping, comparing figures 3.1, 3.2 and 3.3 show the deflection of the beam at various value of the damping coefficient K_D when the speed (V) of the moving load are $V=$ 6.9m/s, 6.6m/s, 6.3m/s. The results show that the amplitude of deflection of the beam gradually decreases as the speed of the load increase. It is found that the gradual decrease in the deflection of the beam is a result of damping coefficient. The analysis of the result shows that for an un-damped Euler-Bernoulli with moving load, the deflection of the beam keeps on increasing, while for Euler-Bernoulli beam with damping coefficient, the deflection of the beam decreases as the speed of the moving load decreases with the various values of damping coefficients. Finally, the result in this work agrees with what obtain in the Literature. Hence the method employed in this work is accurate.

References

[1] Adil E. I. and Fulgence B. R. (2015). Transverse Vibration Analysis of Euler-Bernoulli Beam Carrying Point Mass Submerged in Fluid Media. International Journal of Engineering and Technology; **4** (2).

[2] Carlo S., Tien L. N. and Mohammed H. (2015). An Energy Momentum Method for In-Plane Geometrically Exact Euler-Bernoulli Beam Dynamics'. International Journal for Numerical Methods in Engineering.

[3] Gbadeyan J. A. and Dada M. S. (2001). The dynamic response of plates on Pasternak foundation to distributed moving loads. Nigerian Journal of Mathematical Physics; **5**: 185-200.

[4] George O., Gbeminiyi S. (2015). Analysis of Vibration of Euler-Bernoulli Clamped Laminated beam with non-uniform pressure distribution at the interfaces. European Journal of Engineering and Technology; **3** (2).

[5] Idowu A. S., Are E. B., Joseph K. M. and Daniel S. K. (2013). Dynamic effect of viscous damping on isotropic rectangular plates resting on Pasternak foundation subjected to moving loads. International Journal of mathematics and statistics studies; **1** (2): 12-19.

[6] Isede H. A. and Gbadeyan J. A. (2010). Dynamic Response of Prestressed Tapered Timoshenko Beams to Uniform Partially Distributed Moving loads.

[7] Jinhee Lee (2015): Application of Chebyshev-Tau Method to the free vibration analysis of stepped beams. International Journal of Mechanical Sciences; **101-102**: 411-420.

[8] Kun Y. Y. Jing J. L. and Zhang J. (2015). Stabilization of Euler-Bernoulli Beam Equations with Variable Coefficients under Delayed Boundary output Feedback. Journal of Differential Equations; **2015** (75): 1-14.

[9] Lu S. (2003). An explicit representation of steady state response of a beam on an elastic foundation. Journal of Computers and structures; **57** (1): 151-156.

[10] Maheshwari P., Chandra S. and Basuuhar P. K. (2004). Response of Beams on tensionless extensible geo-synthetic reinforced earth bed subjected to moving loads. Journal of Computers and Geotechnics; **31**: 537-548.

[11] Malik Loudini (2006). Timoshenko Beam Theory based Dynamic Modelling of Lightweight Flexible Link Robotic Manipulators. Encod Nationale Suprieure dInformatique, Algeria.

[12] Mohammed A. R., Al A. R. (2013). Analysis of vibration of a pipeline supported on an elastic soil using differential transform method. American Journal of Mechanical Engineering; **1** (4): 96-102.

[13] Muge B., Metin O. K. and Saglamer A. (2008). Analysis of the Vibration of an Elastic Beam supported on Elastic soil using Differential Transform Method. Archive of Applied Mathematics; **79** (2): 135-146.

[14] Naz R. and Mahomed F. M. (2015). Dynamic Euler-Bernoulli Beam Equation: Classification and Reductions. Journal of Mathematical Problems in Engineering and Technology; **3** (2).

[15] Onu G. (2000). Shear effective in beam finite element on two parameter elastic foundation. Journal of Engineering Mechanics. 1104-1107.

[16] Prasanta K. N. et al (2013). Stabilization of transverse vibration of an inhomogeneous beam' Qscience connect. http://dx.doi.org/10.5339/connect2013-21.

[17] Reza Attarnejad and Ahmad Shabba (2008). Application of Differential Transform Method in Free Vibration Analysis of Rotating Non-Prismatic Beam. World Applied Sciences Journal; **5** (4): 441-448.

[18] Usman M. A. (2003). Dynamic Response of a Bernoulli Beam on Winkler Foundation under the Action of Partially Distributed Load. Nigerian Journal of Mathematics and Application; **16** (1): 128-147.

[19] Usman M. A. (2013). Response analysis of Euler-Bernoulli Beam subjected to Partial Distributed Load. Journal of the Nigerian Association of Mathematical Physics; **25**: 65-76.

[20] Yeih W., Chen J. T. and Cheng C. M. (1999). Applications of dual MRM for determining the natural modes of an Euler-Bernoulli beam using differential transform method.

[21] Zhingang Zhang et al (2015). A Spatial Euler-Bernoulli Beam elements for rigid-flexible coupling: dynamic analysis of flexible structures. Journal of Shock and Vibration; **2015**.

[22] Zingfeng Liu et al (2013). Study on modified differential transform method for free vibration analysis of Euler-Bernoulli Beam. Journal of Structural Engineering and Mechanics; **48** (5).

[23] Usman M. A., and Ogunsan B. T. (2004). Effect of Damping Coefficiets on Euler-Bernoulli Beam subjected to partially distributed load. Unpublished Ph.D. Theses. University of Ilorin, Nigeria.