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HYDROMAGNETIC BOUNDARY-LAYER FLOW OF A NANOFLUID PAST A STRETCHING SHEET EMBEDDED IN A DARCIAN POROUS MEDIUM WITH RADIATION

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Abstract

The problem of laminar fluid flow which results from the stretching of a flat surface in a nanofluid has been obtained using the Adomian Decomposition Method. The model used for the nanofluid was presented in its rectangular form. The model is considered in a porous medium and incorporates the magnetic effect, thermal radiation effect and the effect of Brownian motion and thermophoresis. A similarity solution is presented which depends on Darcy number, magnetic effect, inertia coefficient, Prandtl number, Radiation, Lewis number, Brownian motion number and thermophoresis number. In the results presented graphically, it is observed that the Darcy number enhances the velocity, temperature and concentration profile of the fluid.

1. Introduction

Nanofluid belongs to a new class of heat transfer fluids which consist of both base fluid and nanoparticles. The use of additives is a technique applied to enhance the heat transfer performance of base fluids. The thermal conductivity of the ordinary heat transfer fluids is not adequate to meet today's cooling rate requirements. Nanofluids have been shown to increase the thermal conductivity

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and convective heat transfer performance of the base liquids. Nanofluids are fluids containing nano sized particles. Nanofluids are engineered colloids made of a base fluid and nanoparticles. Nanofluids have novel properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engines. Nanoparticles are of great scientific interest as they are effectively a bridge between bulk materials and atomic or molecular structures. In the past decades, heat transfer enhancement technology has been developed and widely applied to heat exchanger applications; for example, refrigeration, automotives, process industry, chemical industry, etc. Nanofluid coolants showing an improved thermal performance are being considered as a new key technology to secure nuclear safety and economics. The term was coined by Choi (1995). The characteristic feature of nanofluids is thermal conductivity enhancement, a phenomenon observed by Masuda et al. (1993). This phenomenon suggests the possibility of using nanofluids in advanced nuclear systems Aiyesimi et al. (2013). A benchmark study on the thermal conductivity of nanofluids was made by Buongiorno et al. (2009). Venerus et al. (2010) have studied the viscosity measurements on colloidal dispersions (nanofluids) for heat transfer applications. Gharagozloo et al. (2008) have examined the diffusion, aggregation, and the thermal conductivity of nanofluids and Philip et al. (2008) have presented the nanofluid with tenable thermal properties. For the porous medium the Darcy model has been employed. Naturally, the enhancement of thermal conductivity and dispersion of nanoparticles bring about additional thoughts to the heat transfer community that we can use those for a variety of heat transfer applications in terms of heat transfer and thermal management efficiency. Further, Kuznetsov and Nield (2009) have examined the influence of nanoparticles on natural convection boundary-layer flow past a vertical plate, using a model in which Brownian motion and thermophoresis are accounted for. In this pioneering study they have assumed the simplest possible boundary conditions, namely those in which both the temperature and the nanoparticle fraction are constant along the wall. Nield and Kuznetsov (2009) have analyzed the effect of nanoparticles on natural convection boundary-layer flow in a porous medium past a vertical plate and employed the Darcy model for the momentum equation. Bach et al. (2010) have studied theoretically the problem of steady boundary-layer flow of a nanofluid past a moving semi-infinite flat plate in a uniform free stream and it is found that dual solutions exist when the plate and the free stream flow move in the opposite directions. The problem of laminar fluid flow resulting from the stretching of a flat surface in a nanofluid has been investigated numerically by Khan and Pop (2010). Recently, Aiyesimi et al. (2015) carried out an analytical investigation of a convective boundary-layer flow of a nanofluid past a stretching sheet with radiation using the Adomian Decomposition Method. We found it to be appropriate to consider the work of Khan and

Pop (2010) in a porous medium with permeability, and magnetic field effect with Radiation, and use the Adomian Decompositon Method (ADM) to obtain the analytical solution of the model. Aiyesimi et al. (2013) have previously used the Adomian Decomposition to obtain the analytical solution of hydromagnetic boundary layer micropolar fluid flow over a stretching surface embedded in a non Darcian medium with variable permeability. A few examples are the papers by Aiyesimi et al. (2013), Niel et al. (2009), Jiya and Oyubu (2012).
This work is a new development in the literature in which an analytical solution of a magnetohydrodynamic boundary-layer flow of a nanofluid past a stretching sheet embedded in a non Darcian medium with permeability and magnetic field effect with radiation is proposed using the Adomian Decomposition Method.

2. PROBLEM FORMULATION

We consider a steady, two dimensional boundary layer flow of a nanofluid over a continuously moving stretching surface with the linear velocity u(x,y)=ax, where a is constant and x is the coordinate measured along the stretching sheet surface. A uniform magnetic field B_0 is applied along the $y-axis$. We assumed that the stretching surface, the temperature T and the nanoparticle fraction C have constants values T_w and C_w respectively. For this application, we will adopt the formulation of Khan and Pop (2010) in a porous medium with permeability, and magnetic field effect with Radiation and it is governed by the following equations:
Continuity equation:-

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

Momentum equation:-

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\nu\varphi}{k}u-c\varphi u^2-\frac{\sigma B_0^2}{\rho_f}u \tag{2.2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\nu\varphi}{k}v-c\varphi u^2-\frac{\sigma B_0^2}{\rho_f}v \tag{2.3}$$

Energy equation:-

$$\begin{aligned} u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}&=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)\\ +\tau\left(D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\left(\frac{D_T}{T_\infty}\right)\right.&\left.\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)\right) \end{aligned} \tag{2.4}$$

Nanofraction equation:-

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{2.5}$$

Subject to the boundary conditions:

$$\left.\begin{array}{r} y=0: u=ax, \qquad v=0, \qquad T=T_W, \qquad C=C_W \\ y\rightarrow\infty: u\rightarrow 0, \qquad T\rightarrow T_\infty, \qquad C\rightarrow C_\infty \end{array}\right\} \tag{2.6}$$

where u and v are the velocity components along the x and y axes respectively, p is the fluid pressure, φ is the porousity, k is the permeability of the porous media, c is Forchheimer's inertia coefficient, $B_0$is an external magnetic field, ρ_f is the density of the base fluid, σ is the electrical conductivity, α is the thermal diffusivity, $\nu$is the kinematic viscosity, $k^*$is the thermal conductivity, C_p is the specific heat capacity at constant pressure, a is a positive constant, D_B is the Brownian diffusion coefficient, D_T is the thermopheric diffusion coefficient and $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ is the ratio between the effective heat capacity of the fluid with ρ being the density, c is the volumetric volume expansion coefficient and ρ_p is the density of th particles g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, q_r is the radiative heat flux.

Following Roseland approximation we have $q_r=\frac{4\sigma^*}{3\delta}\frac{\partial T^4}{\partial y}$, where σ^* and δ are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. The temperature differences within the fluid is assumed sufficiently small such that T^4 may be expressed as a linear function of Temperature . Expanding T^4 in Taylor's series about T_∞ and neglecting higher order terms, we get

$$T^4 \cong 4TT_\infty^3 - 3T_\infty^4 \tag{2.7}$$

Therefore,

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^*}{3\delta}\frac{\partial^2 T^4}{\partial y^2}$$

Defining the dimensional stream function $(\psi(x,y))$ in the usual way such that $u=\frac{\partial\psi}{\partial y}$ and $v=\frac{\partial\psi}{\partial x}$ and using the following dimensionless variables:-

$$\begin{array}{c} \eta=\left(\frac{a}{\varphi}\right)^{\frac{1}{2}}, \quad \psi=(a\varphi)^{\frac{1}{2}}xf(\eta), \\ \theta(\eta)=\frac{T-T_\infty}{T_W-T_\infty}, \quad \text{and} \quad \chi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \end{array} \tag{2.8}$$

where η, $f(\eta)$, $\theta(\eta)$, $\chi(\eta)$ are the dimensionless fluid distance, velocity profile, temperature profile, and nanoparticle concentration.

An order of magnitude analysis of the y direction momentum equation (normal to the sheet) using the usual boundary layer approximations we have:- $uv, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$, shows that $\frac{\partial p}{\partial y}=0$ substituting the expressions in (2.8) into (2.1)-(2.5) and (2.6) and neglecting the pressure gradient the equations reduces to the following similarity solution:-

$$f''' + f'' - (1+\phi)f'^2 - (D^{-1}a - M)f' = 0 \tag{2.9}$$

$$\left(1+\frac{4R_a}{3}\right)\theta''+prf\theta'+prN_b\chi'\theta'+prN_t\theta'^2=0 \tag{2.10}$$

$$\chi''+L_ef\chi'+\frac{N_t}{N_b}\theta''=0 \tag{2.11}$$

with boundary conditions:-

$$\left.\begin{array}{l} f(0)=0,\quad f'(0)=1,\quad \theta(0)=1,\quad \chi(0)=0 \\ f'(\infty)=0,\quad \theta(\infty)=0,\quad \chi(\infty)=0 \end{array}\right\} \tag{2.12}$$

in which :

$$\left.\begin{array}{l} D^{-1}a=\frac{\nu\varphi}{ak},\quad M=\frac{\sigma B_0^2}{a\rho},\quad \phi=c\varphi x \\ Ra=\frac{4\sigma^*T_\infty^3}{\delta k^*},\quad p_r=\frac{\nu}{\alpha},\quad L_e=\frac{\nu}{D_B} \\ N_b=\frac{(\rho c)_pD_B(C_W-C_\infty)}{(\rho c)_f\nu},\quad N_t=\frac{(\rho c)_pD_T(C_W-C_\infty)}{(\rho c)_f\nu} \end{array}\right\} \tag{2.13}$$

are the inverse Darcy number, Magnetic parameter, inertia coefficient, Radiation, Prandtl number, Lewis number, Brownian motion parameter, and thermophoresis parameter respectively.

3. ANALYSIS OF METHOD

3.1. **Adomian Decomposition Method.** For the purpose of illustrating the method of Adomian decomposition we begin with the (deterministic) form $F(u)=g(t)$ where F is a nonlinear ordinary differential operator with linear and nonlinear term $Lu+Ru$ where we choose L as the highest-ordered derivative. Now L^{-1} is simply $n-$fold integration for an n^{th} order. The remainder of the linear operator is R (in case where stochastic terms are present in linear operator, we can include a stochastic operator term Ru). The nonlinear term is represented b Nu. Thus,

$$Lu+Ru+Nu=g$$

and

$$L^{-1}Lu=L^{-1}g-L^{-1}Nu$$

for initial value problems we convieniently define

$$L^{-1}=\frac{d^n}{dt^n}$$

as the $n-$fold definite integration operation from 0 to t. For the operator $L=\frac{d^2}{dt^2}$, for example we have,

$$L^{-1}Lu=u-u(0)-tu'(0)$$

Therefore,

$$u=u(0)+L^{-1}g-L^{-1}Ru-L^{-1}Nu$$

For the same operator equation but now considering a boundary value problem, we let L^{-1} be an indefinite integral and write $u=A+Bt$ for the first two terms and evaluate A, B from the given condition the three terms are identified as u_0 in the assumed decomposition

$$u = \sum_{n=0}^{\infty} u_n$$

Finally, assuming Nu is analytic, we write

$$Nu = \sum_{n=0}^{\infty} A_n(u_0 \cdot u_n)$$

where the A_n are specially granted Adomian polynomials for the specific nonlinearity.

3.2. **Implementation of Method.** The nonlinear coupled differential equation (2.9) to (2.11) with boundary conditions (2.12) are solved using the ADM methods. If ADM is applied on (9) to (11) and we defined $L_1 = \frac{d^3}{d\eta^3}$ and $L_2 = \frac{d^2}{d\eta^2}$, then

$$L_1[f] = -ff'' + (1+\phi)f'^2 + (D^{-1}a + M)f' \tag{3.1}$$

$$L_2[\theta] = \frac{-3P_r}{(4Ra+3)}(f\theta' + N_b\chi'\theta' + N_t\theta'^2) \tag{3.2}$$

$$L_2[\chi] = -L_e f\chi' - \frac{N_t}{N_b}\theta'' \tag{3.3}$$

Applying inverse operator on equation (3.1) to (3.3), we have

$$L_1^{-1}L_1[f] = -L_1^{-1}[ff''] + (1+\phi)L_1^{-1}[f'^2] + (D^{-1}a + M)L_1^{-1}[f'] \tag{3.4}$$

$$L_2^{-1}L_2[\theta] = \frac{-3P_r}{(3+4Ra)}L_2^{-1}[(f\theta' + N_b\theta'\chi' + N_t\theta'^2)] \tag{3.5}$$

$$L_2^{-1}L_2[\chi] = L_2^{-1}[-L_e f\chi' - \frac{N_t}{N_b}\theta''] \tag{3.6}$$

where $L_1^{-1} = \int\int\int(.)d\eta d\eta d\eta$ and $L_2^{-1} = \int\int(.)d\eta d\eta$
The ADM solution is obtained by:

$$\sum_{m=0}^{\infty} f_m(\eta) = \frac{1 - be^{-(1+M+D^{-1}a+\phi)^{\frac{1}{2}}}}{(1+M+D^{-1}a+\phi)^{\frac{1}{2}}} +$$

$$L_1^{-1}\Big[\sum_{m=0}^{\infty} A_m\Big] + (1+\phi)L_1^{-1}\Big[\sum_{m=0}^{\infty} B_m\Big] + (Da^{-1} + M)L_1^{-1}\Big[\sum_{m=0}^{\infty} f'_m\Big] \tag{3.7}$$

$$\sum_{n=0}^{\infty} \theta_n(\eta) = ce^{-\eta} +$$

$$\Big(\frac{-3P_r}{(3+4Ra)}\Big)\Big(L_2^{-1}\Big[\sum_{n=0}^{\infty}C_n\Big]+N_bL_2^{-1}\Big[\sum_{n=0}^{\infty}D_n\Big]+N_tL_2^{-1}\Big[\sum_{n=0}^{\infty}E_n\Big]\Big) \tag{3.8}$$

$$\sum_{n=0}^{\infty}\phi_n(\eta)=he^{-\eta}-L_eL_2^{-1}\Big[\sum_{n=0}^{\infty}F_n\Big]-\frac{N_t}{N_b}L_2^{-1}\Big[\sum_{n=0}^{\infty}\theta''_n\Big] \tag{3.9}$$

where

$$A_m=\sum_{v=0}^{m}f_{m-v}f''_v \tag{3.10}$$

$$B_n=\sum_{v=0}^{n}f'_{n-v}f'_v \tag{3.11}$$

$$C_n=\sum_{v=0}^{n}f_{n-v}\theta'_v \tag{3.12}$$

$$D_n=\sum_{v=0}^{n}\theta'_{n-v}\chi'_v \tag{3.13}$$

$$E_n=\sum_{v=0}^{n}\theta'_{n-v}\theta'_v \tag{3.14}$$

$$F_n=\sum_{v=0}^{n}f_{n-v}\chi'_v \tag{3.15}$$

In other to take care of problems at infinity, we therefore take functions which satisfies the boundary conditions at infinity as our initial guesses and introduced auxiliary constants b, c, and c in other to maintain the initial condition.
For determination of other components of $f(\eta)$, $\theta(\eta)$ and $\chi(\eta)$, we have:

$$\sum_{m=0}^{\infty}f_{m+1}(\eta)=$$

$$-L_1^{-1}\Big[\sum_{m=0}^{\infty}A_m\Big]+(1+\phi)L_1^{-1}\Big[\sum_{m=0}^{\infty}B_m\Big]+(Da^{-1}+M)L_1^{-1}\Big[\sum_{m=0}^{\infty}f'_0\Big] \tag{3.16}$$

$$\sum_{n=0}^{\infty}\theta_{n+1}(\eta)=$$

$$\Big(\frac{-3P_r}{(3+4Ra)}\Big)\Big(L_1^{-1}\Big[\sum_{n=0}^{\infty}C_n\Big]+N_bL_2^{-1}\Big[\sum_{n=0}^{\infty}D_n\Big]+N_tL_2^{-1}\Big[\sum_{n=0}^{\infty}E_n\Big]\Big) \tag{3.17}$$

$$\sum_{n=0}^{\infty}\chi_{n+1}(\eta)=-L_eL_2^{-1}\Big[\sum_{n=0}^{\infty}F_n\Big]-\frac{N_t}{N_b}L_1^{-1}\Big[\sum_{n=0}^{\infty}\theta_n''\Big] \tag{3.18}$$

where b, c, and h are all constants to be determined for actual solutions.
The general solutions are:

$$f(\eta)=\sum_{m=0}^{\infty}f_m(\eta)=f_0+f_1+f_2\cdots \tag{3.19}$$

$$\theta(\eta)=\sum_{m=0}^{\infty}\theta_m(\eta)=\theta_0+\theta_1+\theta_2+\cdots \tag{3.20}$$

$$\chi(\eta)=\sum_{m=0}^{\infty}\chi_m(\eta)=\chi_0+\chi_1+\chi_2+\cdots \tag{3.21}$$

for conveniences, we used Maple-18 to compute the integrals.

Table 3.1 Comparison between Numerical method and the present work for $f''(0)$ at ϕ=1

M	Da	Numerical	Present work
0.1	1	1.6702706590802672	1.85678160
1	1	1.9169861441991335	2.07706528
10	1	3.5587173882820053	3.638850095
30	1	5.7154007886807818	5.766624927
1	0.1	3.5587173882820053	3.638850094
1	1	1.9169861441991634	2.077065282
1	10	1.6702706590802672	1.856781605
1	30	1.6507085596419419	1.839644225

4. RESULTS AND DISCUSSION

The nonlinear coupled differential equations (2.9) to (2.11) with boundary conditions (2.12) are solved using the Adomian Decomposition Methods. In order to assess the accuracy of the present method, we have compared our solution for for different values of Magnetic parameter (M), and Darcy number (Da) at with the numerical method (shooting technique) as shown in Table 3.1.
It was observed that the present method is in good agreement with the numerical method except for $M = 1$ and $Da = 1$.

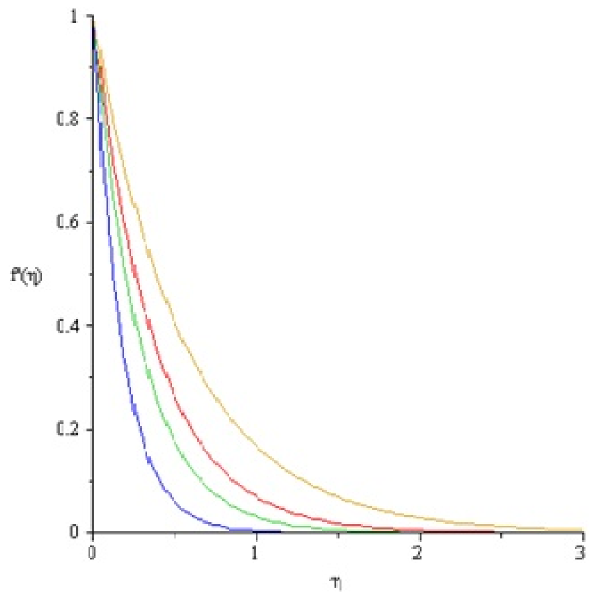

Figure:4.1 Effect of Magnetic parameter on the velocity profile

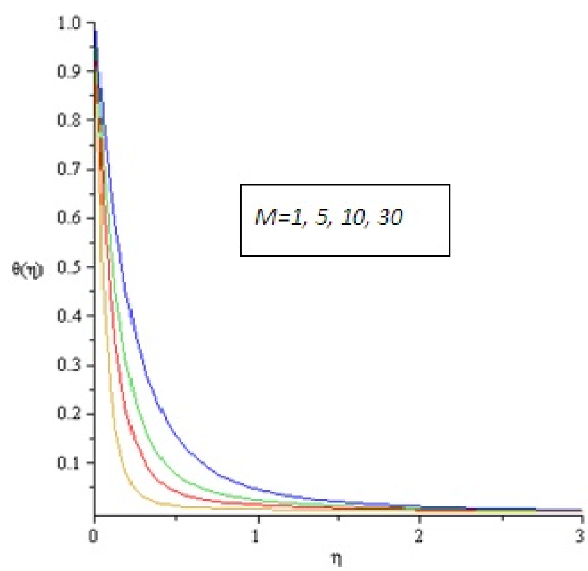

Figure:4.2 Effect of Magnetic parameter on temperature profile.

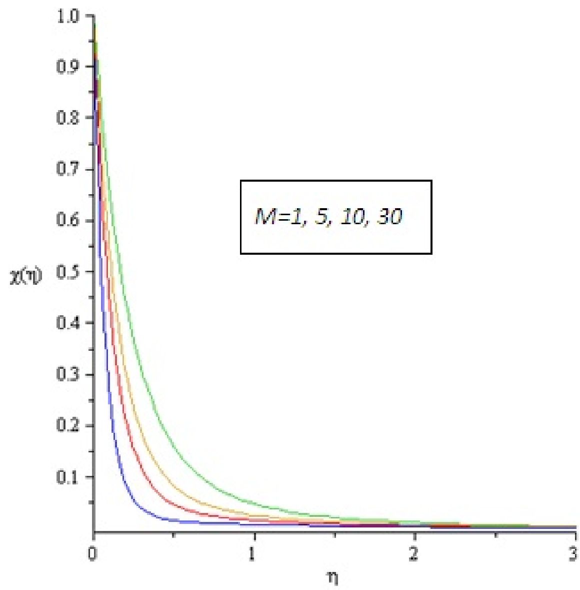

Figure:4.3 Effect of Magnetic parameter on concentration profile

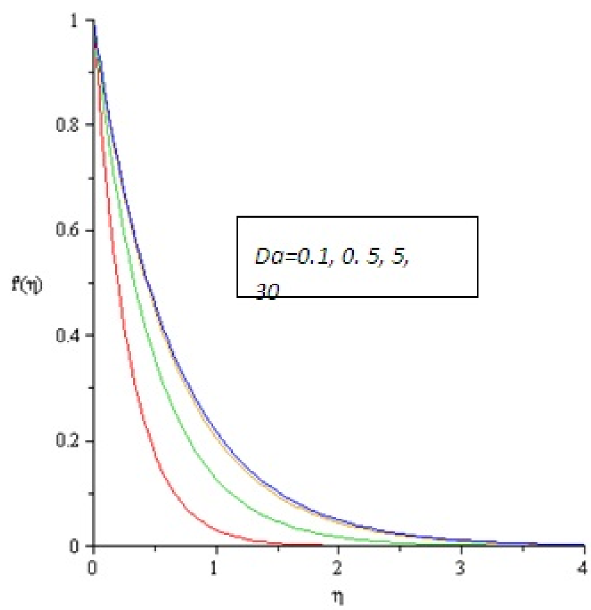

Figure:4.4 Effect of Darcy number on velocity profile.

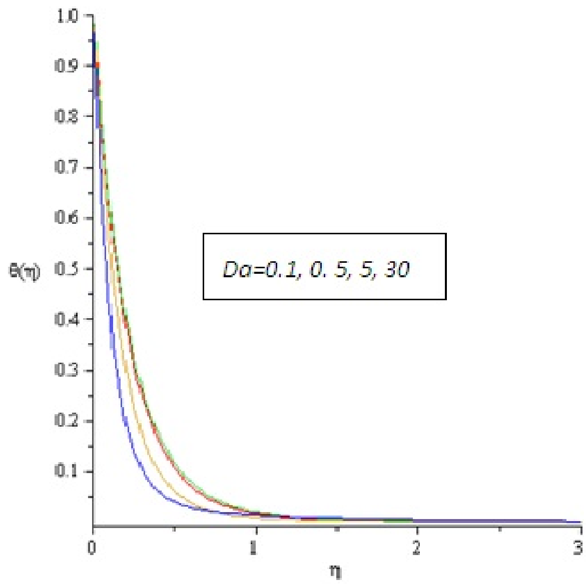

Figure:4.5 Effect of Darcy number on temperature profile.

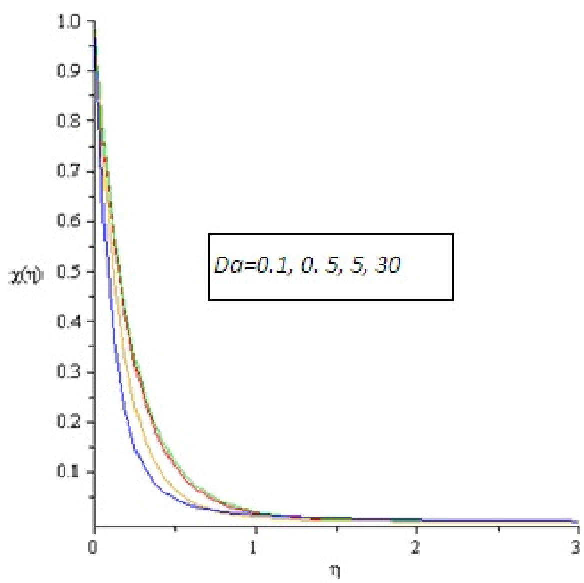

Figure:4.6 Effect of Darcy number on concentration profile

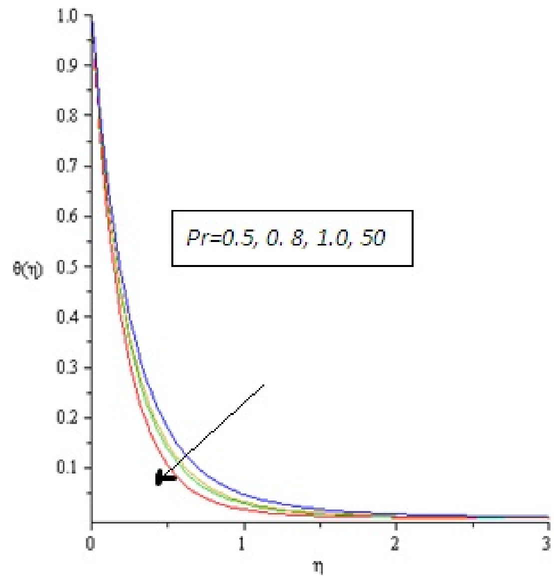

Figure:4.7 Effect of Prandtl number on temperature profile

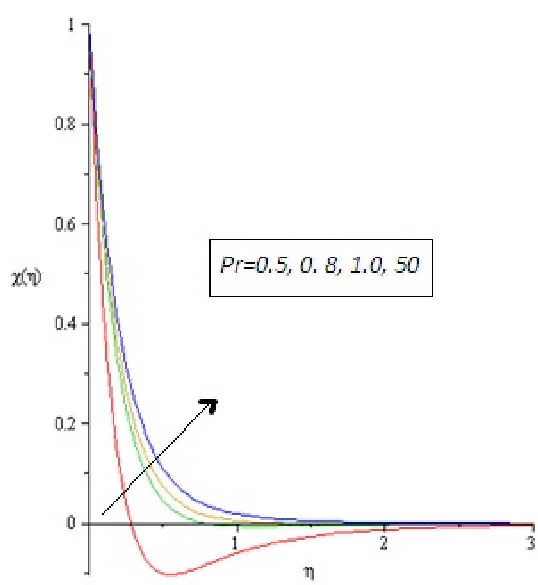

Figure:4.8 Effect of Prandtl number on concentration profile

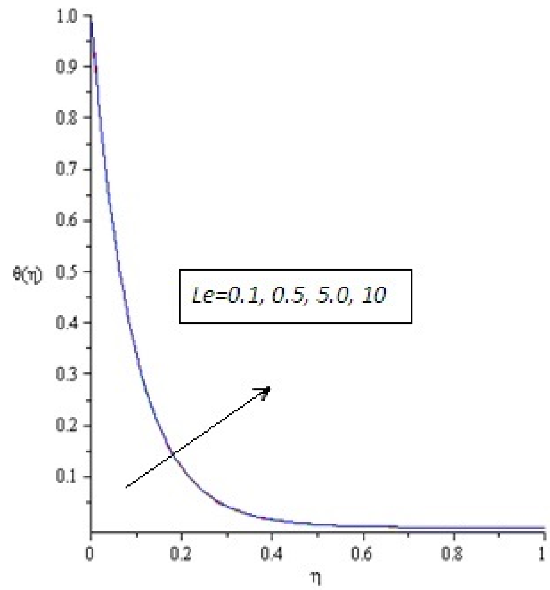

Figure:4.9 Effect of Lewis number on temperature profile

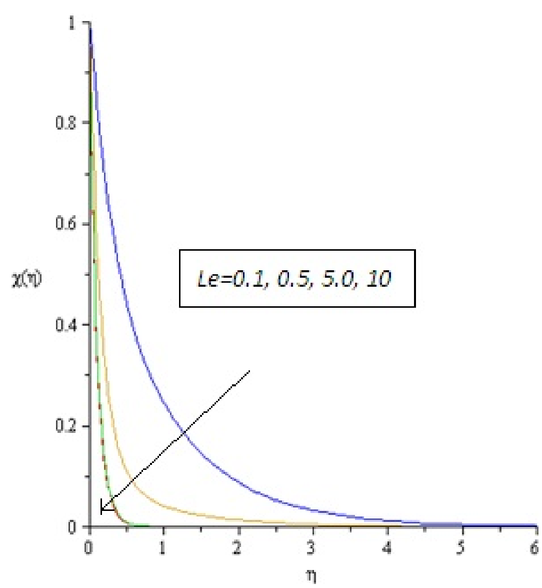

Figure:4.10 Effect of Lewis number on concentration profile

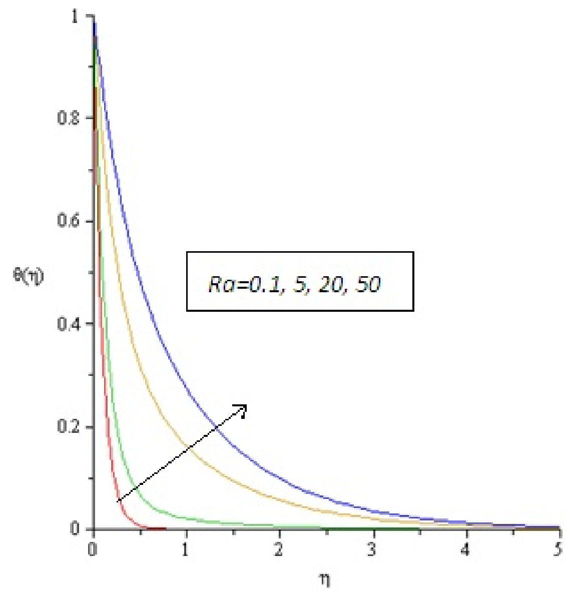

Figure:4.11 Effect of Radiation on temperature profile

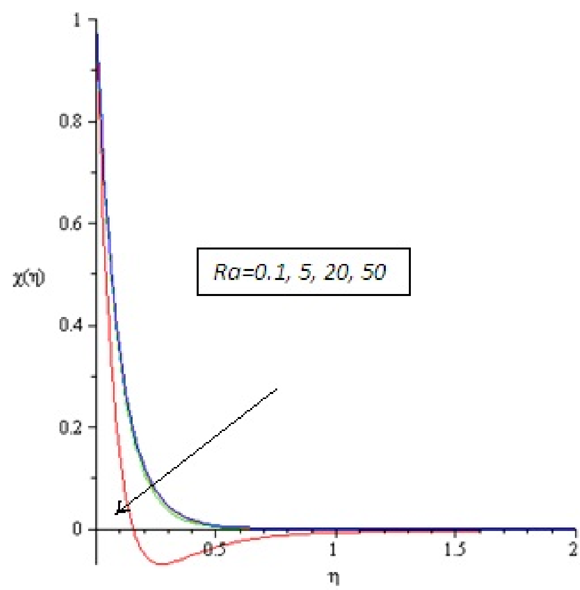

Figure:4.12 Effect of Radiation on concentration profile

Figures 4.1 to 4.3 shows the effect of magnetic parameter on velocity, temperature, and concentration profiles in a porous medium. It is observed that as the magnetic parameter increases, the velocity, temperature and concentration profile reduces. Reduction in the velocity profile implies that the fluid under consideration becomes more viscous which leads to reduction in the temperature of the fluid and consequently decrease in the nanofraction concentration. Figures 4.4 to 4.6 depict the effect of Darcy number on the velocity, temperature and concentration profile. It is observe from the figures that when Darcy number is less than one (ie permeability is less than the squire diameter of the fluid particles), the fluid is laminar while it is turbulent if other wise (ie permeability is greater than the squire diameter of the fluid particles) in the porous medium. Increase in the Darcy number enhances both the fluid temperature and concentration profile.

Figures 4.7 to 4.8 present the effect of Prandtl number on the temperature distribution and nanofraction concentration. It is observe that as prandtl number increases the thermal boundary layer thickness reduces which enable heat to diffuse away from the system for higher values of prandtl number. The nanofraction concentration is enhancing as prandtl number increases.

Figures 4.9 to 4.10 shows that the Lewis number has no significant effect on the temperature profile but enhances the nanofraction concentration as it increases in the porous medium.

Figures 4.11 to 4.12 shows the effect of Radiation on temperature and nanofraction concentration profile. As the radiation parameter increase the thermal boundary thickness increases while the nanofraction concentration reduces within the porous medium.

5. CONCLUSION

The problem of laminar fluid flow resulting from the stretching of a flat surface of a nanofluid in a porous medium with magnetic effect and radiation has been obtained using the Adomian Decomposition Method for the first time. The model used for the nanofluid was presented in its rectangular coordinate system and incorporates the effect of Brownian motion, and thermophoresis parameter. A similarity solution was presented which depends on the Darcy number, magnetic parameter, coefficient of inertia, Prandtl number, Lewis number, Brownian motion, thermophoresis number, and radiation. It was found that:-

(1) The fluid becomes more viscous as the velocity profile reduces due to increase in the magnetic parameter; which leads to reduction in the fluid temperature.
(2) When Darcy number is less than one, the flow is laminar and turbulent if other wise.
(3) Smaller values of Prandtl number are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the

heated surface more rapidly for higher value of Prandtl number. Hence there is a reduction in temperature with increase in the Prandtl number.

(4) The inertia coefficient was also kept constant throughout this work.

(5) The result for the skin friction coefficient was in good agreement with that of the numerical method.

(6) All the graphs presented in this work satisfies the boundary conditions, which further proved the efficiency of this method of ADM.

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